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XLIV. *Twisted Strips.*

By Professor JOHN PERRY, *F.R.S.**

IN a paper read at the British Association Meeting this year, and also at a former meeting of the Physical Society, I referred to the curious behaviour of a twisted strip. The twisted strip must not be confounded with what has been called the Ayrton and Perry Spring. It is a straight strip of metal to which a permanent twisted appearance has been given, a twisting moment being applied about the axis of the strip to give it the permanent twist, care being taken to prevent the axis of the strip from lateral motion. In fact the axis remains straight, and is the axis of all the spiral lines now formed by the lines which were originally straight and parallel to the axis. When an axial force is employed to elongate a twisted strip, a slight elongation takes place accompanied by a large relative rotation of the ends of the strip. The only simple hypothesis on which, as I told the Society on a former occasion, I could see my way to building a possible theory was to consider in any very short length of the strip any two spiral filaments equally distant from the axis as two threads in a bifilar suspension. Under the action of any axial force these two threads tend to produce untwisting, and the sum of

* Read November 1, 1889.

all these untwisting moments is equal to the torsional rigidity of the cross section multiplied by the angle of untwist.

The phenomena are so complicated that I never hoped to explain them all by this or any other hypothesis. I was seeking for a roughly correct theory only.

The breadth being b and thickness t, we may roughly assume that the section remains rectangular; and as b is always more than ten times t, we may take the torsional rigidity as $\frac{N}{3}bt^3$. If then θ is the angular amount of untwisting produced per unit length of the strip, the moment of torsional resistance is

$$\frac{N}{3}bt^3\theta.$$

Let w be the axial force. If ϕ is the twist of the strip per unit length, then for a filament whose horizontal section is $t.\delta x$, at the distance x from the axis (as t is small compared with b I take the filament as extending quite across the section), if p is the vertical tensile stress on the section, it is obvious that the part of the untwisting moment due to one filament is $tpx^2\phi.\delta x$; so that

$$\frac{N}{3}bt^3\theta = t\phi\int_{-\frac{1}{2}b}^{\frac{1}{2}b} px^2.dx; \quad . \quad . \quad . \quad . \quad (1)$$

also

$$w = t\int_{-\frac{1}{2}b}^{\frac{1}{2}b} p.dx. \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

I explained on the former occasion that, for a working theory, it ought to be sufficient to take p constant over the section. Hence, from (2),

$$p = \frac{w}{bt}; \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

and inserting this value in (1), we have

$$\frac{N}{3}bt^3\theta = \frac{1}{12}t\phi b^3\frac{w}{bt},$$

or

$$\theta = \frac{\phi bw}{4t^3N}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

If θ is small in comparison with ϕ, it is allowable to take ϕ

as the twist of the strip when no axial load is acting—that is, its permanent twist.

Now I announced on the former occasion that careful experiments made upon large twisted steel strips of various dimensions did not by any means agree with my result. It is true that θ was proportional to w; but it seemed rather independent of ϕ, and instead of being proportional to b, θ seemed to get less as b was greater; and, again, instead of t^3 in the denominator some power of t between 2 and 3 ought to be taken.

Under these circumstances I asked members of the Society for assistance in obtaining a working theory, but hitherto I have not obtained any.

I wish now to announce that the experiments I referred to, made very carefully by Mr. Still, of the Finsbury College, under my direction, were not sufficiently exhaustive. Mr. C. E. Holland has lately been kind enough to make an experiment for me on a very small strip of platinum silver. He finds that when ϕ is large his results agree very closely with the former results; but as ϕ is made less and less, the law connecting θ and ϕ becomes more and more nearly what is given by (4). The experiments were carried out upon double-twisted strips. That is, a straight strip was taken with its ends fastened in a frame; it was caught in the middle and there a twisting-couple was applied, so that one half of it took a right-handed and the other a left-handed permanent twist. Beginning from almost the condition $\phi=0$, by giving more and more permanent twist to the strip and applying loads in each case, it was seen that, for a given axial load, θ was nearly proportional to ϕ at first, but as ϕ became greater and greater the increase of θ was less rapid, θ eventually remaining nearly constant, although ϕ was increased.

I now know that Mr. Still's experiments were carried out upon strips in which there existed so much permanent twist as to resemble the strips in the condition in which they were towards the end of Mr. Holland's twisting operations, and hence θ seemed independent of ϕ.

Now if, instead of taking p constant and using (3), we take p as getting less for filaments at greater distances from the axis, it is obvious that we shall arrive at a result which,

although more complicated than (4), will represent much better the results of experiments.

Thus, as is probable enough, if the tensile strain in every filament is such as to keep a section at right angles to the axis parallel to itself, it is easy to show that, instead of p being constant, we must take

$$p = \frac{k}{t(1+x^2\phi^2)}. \quad \ldots\ldots \quad (5)$$

Using this in (2), we find

$$k = \frac{\phi w}{2\tan^{-1}\frac{1}{2}b\phi}.$$

Substituting (5) in (1) we find, on simplifying,

$$\theta = \frac{3w}{Nt^3}\left\{\frac{1}{2\tan^{-1}\frac{1}{2}b\phi} - \frac{1}{\phi b}\right\}. \quad \ldots \quad (6)$$

It is obvious that, except when $b\phi$ is great, (6) becomes

$$\theta = \frac{w}{4Nt^3}\left\{\phi b - \frac{\phi^3 b^3}{15} + \frac{\phi^5 b^5}{115}\right\}, \quad \ldots \quad (7)$$

which agrees with (4) for small values of ϕb.

And evidently from (6), as $b\phi$ gets greater and greater, θ reaches more and more nearly the limiting value,

$$\frac{3w}{Nt^3\pi}.$$

Making p any other likely function of $x\phi$ than that given in (5) willmerely modify the constants in (7).

Professor Ayrton and I have used these strips in weighing-machines, instruments in which forces require to be measured, and instruments in which small motions require to be magnified. We are now using the double-twisted strip of constant length, but with initial pull in it, as a thermometer and as a galvanometer. It is my opinion that it only requires to be better known to be largely used; and although I seem to have spent rather in vain a considerable amount of time in trying to get a working theory, I do not think the time has been really wasted. It is no mere curious puzzle, it is a problem of practical importance.

The two great difficulties are—(1) What is the law of p over the section? (2) To what extent is it wrong to regard the

section as resisting untwisting with the moment

$$\frac{N}{3} bt^3\theta ?$$

that is, to what extent has the original twisting given initial shear strains to the material?

XLV. *On a Carbon Deposit in a Blake Telephone Transmitter.* *By* F. B. Hawes*.

(Abstract.)

The Author exhibited photographs of the interior portions of the transmitter on which the deposit had taken place. These portions consist of a metal diaphragm, a highly polished carbon button, and a platinum contact-piece carried by a German-silver spring placed between them. The diaphragm presented a mottled appearance, due to the deposit, but the part which had been behind the German-silver spring seemed comparatively clean. The deposits on the carbon button and German-silver spring were much less dense than that on the exposed parts of the diaphragm, and the space near the point of contact between the platinum and carbon was free from deposit. The deposit was fairly adherent, considerable rubbing being necessary to remove it, and, on examination under the microscope, particles of copper and metallic crystals could be seen. The author believes the deposit due to some kind of bombardment of carbon particles, but was unable to say why it should occur, or why the varnished diaphragm should receive the greater deposit, although it was further from the carbon than the German-silver spring.

Mr. C. V. Boys said the photographs reminded him of a phenomenon he observed some time ago on a glass sheet against which one terminal of a dry pile had been resting for some weeks. Just as on the carbon button, the glass near the point of contact was clean, and has a comet-shaped deposit formed around it. He could offer no explanation of the appearance.

* Read February 21, 1890.

XLVI. *On an Apparatus for the Distillation of Mercury in a Vacuum.* *By* Wyndham R. Dunstan, *M.A., F.C.S., Professor of Chemistry to the Pharmaceutical Society of Great Britain, and* T. S. Dymond, *F.I.C., F.C.S.**

[From the Research Laboratory of the Pharmaceutical Society.]

Distillation of impure mercury constitutes the best method of removing foreign metals, and distillation in a vacuum is the only feasible plan of conducting the operation in the laboratory. It is doubtful whether mercury can be completely freed from zinc by distillation, and it is therefore safer to remove this metal, before distillation is resorted to, by agitating the mercury with warm concentrated hydrochloric acid.

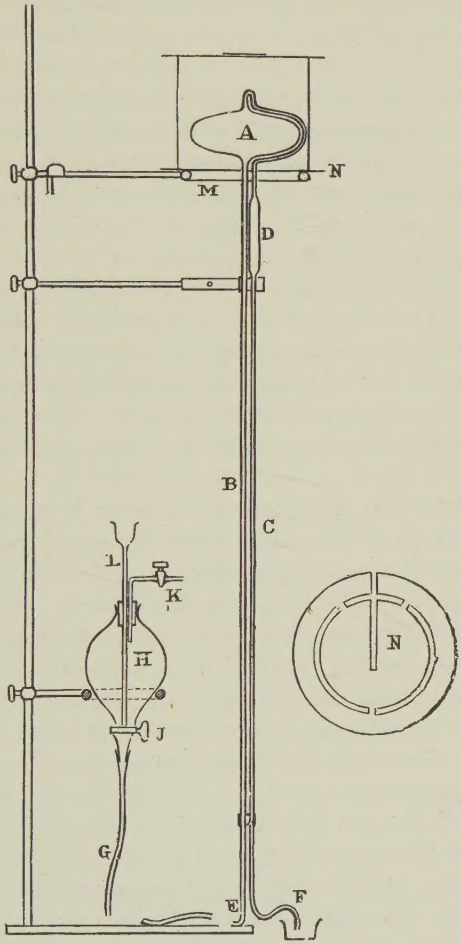
Different forms of apparatus for distilling mercury in a vacuum have been described by Weinhold, Weber, Wright, and Clark. The action of these different pieces of apparatus is primarily dependent on the same principle; but that described by Clark (Phil. Mag. 1884, xvii. p. 24) is undoubtedly the most convenient and is the form which is ordinarily used. It possesses, however, several disadvantages. It is inconveniently long, being at least 5 feet in length, and needs a special place in the laboratory where it must be permanently fixed. Moreover the apparatus is only applicable when a comparatively large quantity of mercury is available, since at the conclusion of the operation there remains in the apparatus between one half and one quarter of a litre of undistilled mercury, the amount being chiefly dependent on the diameter of the distillation-tube. To this residue more mercury must be added before distillation can be recommenced.

The still we are about to describe does not entail these disadvantages. It is easy and inexpensive to construct, and when once started is automatic in its action. It is rather more than a metre long, and when required for use can immediately be clamped on a retort-stand and may be as readily dismounted again. The residue of undistilled mer-

* Read March 7, 1890.

cury which remains in the apparatus is less than 20 cubic centimetres.

The apparatus is constructed in the following manner:—A bulb A is blown on one end of a piece of thick-walled soft glass tubing B, and is made to assume somewhat the shape of



an oblate spheroid, so that nearly the whole of the mercury may be distilled out of it. The other end of the tube has a small elbow-bend to admit of its connexion with the reservoir H by means of the stout rubber-tubing G. The diameter of the bore of this glass tube is 3 millim., and its length 1 metre,

measured from the point at which it joins the bulb to the bend at the other end. To the summit of the bulb is sealed a piece of tubing which is bent round one side of the bulb and terminates in the limb C, which acts as a Sprengel pump. The length of the tube from the lower surface of the bulb to the bend at its other end is about 1 metre, and the bore has a diameter of 1·5 millim. The upper part of this limb, at a distance of about 5 centim. from the bulb, is expanded to form a chamber D for the condensation of the mercury vapour; this chamber is 1 decim. long and 1 centim. wide. The other end of the limb is for convenience bent twice at right angles F, and beneath it is placed a receptacle for the distilled mercury. The movable reservoir H, attached by means of at least 1 metre of stout rubber-tubing, known as "pressure" tubing, to E is a large tap-funnel having a capacity at least twice as great as that of the bulb A. It is provided with a tightly-fitting indiarubber cork carrying a small tapped tube K, which projects just below the cork, and a thistle-funnel L which extends to the bottom of the reservoir. The apparatus is supported by a clamp immediately below the chamber D, and it rests on the bend E of the stout limb B.

Before the still is set in action it must be thoroughly dried. The reservoir H is then filled with dry mercury and raised until the liquid has ascended to the summit of the bulb A, and has commenced to fall over into the limb C. The reservoir is now slightly lowered, and by means of the tap J the flow of mercury is checked until it falls only moderately fast (as in a Sprengel pump) down the limb C. In about five minutes the chamber D will have been exhausted of air. As soon as the sharp "click" of the falling mercury is heard and the descending column in C is almost continuous, the reservoir is lowered until the bulb A is about half full. The reservoir H is now adjusted to act automatically in maintaining a constant level of mercury in the bulb A. The stopcock J having been shut, the bulb is heated and the distillation allowed to proceed for about half an hour until the vapour-pressure of the mercury for the maximum temperature to which the bulb will be raised has been attained. Air is then sucked out of the tube K until bubbles pass up through the mercury, when the tap in K is instantly closed. The tube of the funnel L now con-

tains no mercury, the pressure of the air and of the mercury in the reservoir being exactly equal to the atmospheric pressure acting through L. The level of the mercury in the reservoir must now be adjusted until the surface of the mercury in the bulb is distant from the bottom of the reservoir (above J) by the height of the barometric column less the vapour-pressure of mercury for the temperature at which distillation occurs. This temperature having been ascertained, the adjustment may be made after direct measurement of the mercurial column. It may also be effected perhaps more readily by raising the reservoir little by little until on momentarily opening the stopcock J air is observed to bubble through the mercury. As soon as this occurs, the stopcock J is left open. The mercury from the reservoir H will now be automatically supplied to the bulb A. As the mercury distils out of the bulb more rises from the reservoir to take its place, and air enters from the funnel L and bubbles through the mercury in the reservoir into the closed space above. Through this action of the reservoir the level of the mercury in the bulb will remain the same until the whole of the mercury contained in the reservoir has been distilled.

In order to heat the bulb A the tubes just below it are surrounded by the ring-burner M, which may be a circular piece of metal or glass tubing perforated at regular intervals with small holes; the flames proceeding from these perforations should be very small, and must on no account be allowed to impinge on the bulb or on either of the tubes. The bulb is now enclosed on all sides so that a hot air-chamber may result. This is best done by cutting a circular disk N, whose circumference is somewhat greater than that of the ring-burner, out of asbestos millboard, incisions being made to correspond with the tubes so that the disk may slide under the bulb and rest on the burner itself. On this disk as a base a cylinder of the same material, or of glass, is placed, the cylinder being wider and taller than the bulb. The cylinder is closed at the top by a rather larger and circular piece of asbestos millboard, in the centre of which a small round hole should be cut. By partially or entirely covering this hole with a small disk of millboard, it is easy to regulate the admission of air to the chamber. The cylinder

is prepared from the millboard by thoroughly wetting it and rolling it round a bottle or wooden block of the proper size. When dry the cylinder is removed and secured by twisting a piece of thin copper wire round it. The bulb is thus heated in the air-chamber, and not by direct contact with the flames, and the bulb is effectually protected from the disastrous effects of draughts of cold air.

The temperature of the air-bath when distillation is proceeding ranges between 200° and 300° C. Since fresh mercury is automatically supplied to the bulb the still requires no attention, and may safely be left at work day and night provided that the gas pressure is almost constant. This may of course be ensured by fixing a gas-regulator in the air-bath. We have found, however, that an ordinary Sugg's dry governor introduced between the gas supply and the burner answers the purpose sufficiently well.

When the still is started for the first time the air-film on the glass is detached and passes away down the limb C. The distillation consists in continuous evaporation from the liquid surface unaccompanied by actual ebullition, or by the "bumping" and spirting which are so noticeable when mercury is distilled under the ordinary atmospheric pressure.

The rate of distillation is mainly dependent on the capacity of the bulb A. We have generally worked with a bulb rather smaller than that described above, because it is easier to blow it of sufficient thickness. As the still is automatic in its action and needs no attention the quick distillation of a large quantity of mercury is not a point of much importance. It has frequently happened that the distillation has come to an end during the night, both the reservoir and the bulb being discovered nearly if not quite empty. This, however, has occasioned no inconvenience; it is only necessary to introduce more mercury into the reservoir and from thence into the bulb. This can be done without extinguishing the burner. The stopcock J is closed and the tap K is opened. Mercury is then poured through the funnel L. The stopcock F is now opened very gradually, so that cold mercury may not suddenly rush against the hot glass, until the bulb is half full, when the stopcock J is closed again. A piece of rubber-tubing is now attached

to K, and air is sucked out until bubbles begin to rise through the mercury in the reservoir, when K is shut; after the lapse of about half an hour the stopcock J is opened. When a distillation has been finished and the apparatus is not further required for the present, it may be left standing with the stopcock J closed, and the end F dipping into mercury or otherwise sealed; it can then be started again at any moment. To empty the still, F is closed and the apparatus is carefully tilted so that the mercury remaining in C may flow into the chamber D. If now the end F be cautiously unclosed air will enter the apparatus through the empty limb C.

When this apparatus is at work in a dark room the remarkable phenomenon of the "flashing" of mercury is observed. It is seen in the condensation-chamber D. As the mercury liquefies flashes of green light are produced, which exhibit the spectrum of mercury. It would seem that this is an electrical effect arising from friction between mercury and glass. We have noticed that it is enormously intensified when one of the dischargers of a Wimshurst machine is brought near the chamber D. Under these circumstances the entire chamber becomes illuminated with a magnificent green light.

The apparatus may also be conveniently employed for observing the electrical discharge through mercury-vapour at different temperatures. For this purpose the ascending column of mercury in B and the descending column of mercury in C are used as electrodes. One terminal from the coil dips into the mercury in the reservoir H, the other into the distilled mercury which has collected in the receptacle under F, and below the surface of which the extremity of the tube C is immersed.

We are now making observations on the character of the discharge which occurs under these conditions, in the absence of any foreign metal.

Prof. Thompson said he devised a simple form of distilling-apparatus some time ago which answered fairly well, and could be made by any amateur glass-worker. It consisted of a double barometer, one leg of which was of small bore, so as to act as a Sprengel-tube. The rising part of the bend at the

top of the larger tube was expanded, and served as the evaporating-chamber, below which a burner was placed.

The President asked why Clark's apparatus is made so lengthy.

In reply to this question Mr. Boys said that, as the fall-tube goes down within the rising one, the mercury near the top of the latter is heated by the condensing mercury (thus economising gas), and hence condensation does not take place until the vapour has passed a considerable distance down the fall-tube.

XLVII. *The Theory of Osmotic Pressure and its bearing on the Nature of Solutions.* *By* SPENCER UMFREVILLE PICKERING, *M.A.**

No one can doubt the mathematical correctness of the conclusions which Arrhenius, van't Hoff, Ostwald, and others draw from the premisses with which they start in their arguments respecting osmotic pressure, nor can we doubt the value of connecting numerous actions with one and the same cause, or that there are a large number of instances in which the observed facts are in substantial agreement with their conclusions. But we may, I think, legitimately doubt whether the premisses of the arguments are sound, whether the conclusions harmonize as well as they should with experimental data, whether the theory is more than a mathematical exercise, or more than a convenient working hypothesis of a rough character, instead of being, as its supporters maintain, an hypothesis established so firmly that we may build upon it a physical theory of solution.

The direct measurement of osmotic pressure has been made in but a few cases, and those cases are ones in which the substances examined are eminently unfitted for showing the presence of any chemical action which may be present. Of the phenomena correlated with osmotic pressure, the lowering of the freezing-point of a solvent by the addition of foreign substances is the one which has received the greatest attention, thanks to Raoult's classical work, and is the one which forms the main support of the theory.

* Read March 7, 1890.

In examining the experimental facts the following questions must be asked, and if the theory is correct they must be answered in the affirmative.

(1) Is the molecular depression—using this term to mean the lowering of the freezing-point produced by one foreign molecule on 100 *molecules* (not *parts by weight*) of the solvent—a constant, whatever the foreign substance is, provided that the amount of the latter is not greater than that which would give a pressure equal to that produced by it in the gaseous state at the same temperature?

(2) Are such deviations from regularity, as are observed in exceptional cases, in the direction in which they should be according to the theory?

(3) Is the depression a constant whatever the solvent is?

(4) Is the depression constant when the proportions of the solvent and dissolved substance are varied within the limits mentioned in (1)?

(5) When stronger solutions are taken, is the deviation from regularity in the direction in which it should be according to the theory?

(6) Is the deviation always in the same direction, and always regular?

I believe that I can show that the answer to every one of these questions is an unqualified No.

The main facts of the case as elucidated by Raoult's work are that, when various substances are dissolved in

Benzene,
Nitrobenzene,
Ethylene dibromide,
Formic acid,
or Acetic acid,

the molecular depression is about 0°·63, though there are a few substances in each case which give an abnormal depression of about half this amount.

With water the molecular depression is generally 2°·06, but in many cases approximates to 2°·61, values which Raoult takes to be three and four times 0°·63 (1°·89 and 2°·52) respectively. Abnormally low values are in the same proportion to the normal ones as with other solvents; they are about 1°·03 (half of 2°·06), and are obtained with some

inorganic substances, and with the majority of organic substances at present investigated.

All these determinations apply to solutions containing about 1 molecule to 100 molecules of the solvent, and are, therefore, three or four times stronger than they should be in order to be comparable with gases under ordinary conditions. We shall have to decide whether the discrepancies which they show are such as could be reasonably attributed to increasing the pressure from one to three or four atmospheres.

Influence of the Nature of the Dissolved Substance.

The half values obtained in many cases are explained by the duplication of molecules, owing to the solutions not being sufficiently dilute; that is, an additional pressure of two or three atmospheres will cause the vapour-densities of hydrochloric acid *, methyl, ethyl, amyl, and butyl alcohols †, benzol, and many other substances to double themselves (assuming that they retain their gaseous condition). This may be so, but direct determinations cannot be, or have not been, made.

Leaving these abnormal values for the present, we find that the normal ones exhibit the following variations:—

Solvent.	Number of instances investigated.	Variation in the value of the depression ‡.
Acetic acid	57	30 per cent.
Formic acid	9	14 „
Benzene	41	12 „
Nitrobenzene	13	7 „
Ethylene dibromide	5	2 „
Water	59	60§ „
„ (with organic substances)	29	40 „

These variations appear somewhat too large to be attributed to the fact that we are working at two or three more atmospheres of pressure than we should be.

* Solvent, Acetic acid (Raoult, *Ann. Chim. Phys.* [6] xi. p. 72, from which paper this and other values quoted here are taken).

† Solvents, Benzene, &c.

‡ Percentage on the lowest value.

§ Omitting borax, where the value is exceptionally high.

It is noticeable that the organic solvents, except acetic acid, give comparatively constant results, but that, as soon as we come to the inorganic solvent water (which we know shows a much greater tendency to form "molecular" compounds than the other liquids), we get a much larger variation, and the following values * will justify the assumption that when other solvents of a similar nature are examined, still greater variations will be observed.

Solvent.	Dissolved substance.	Depression†.	Variation†.
		°	Per cent.
Tetrahydrate of sulphuric acid.	Water.	0·08	100
Do.	Sulphuric acid.	0·04	
Monohydrate of sulphuric acid.	Water.	0·02	100
Do.	Sulphuric acid.	0·01	
Sulphuric acid.	Water.	0·07	500
Do.	Sulphuric anhydride.	0·42	
Hexahydrate of calcium chloride.	Water.	0·003	233
Do.	Anhyd. calc. chlor.	0·010	
Tetrahydrate of calcium nitrate.	Water.	0·007	100
Do.	Anhyd. calc. nitrate.	0·015	

As an instance of a similar character we may quote Heycock and Neville's determinations of the depression of the freezing-point of sodium by various metals ‡.

Metal added.	Molec. depression.
	°
Gold	4·5
Thallium	4·3
Mercury	4·5
Cadmium	3·0
Potassium	3·6
Lithium	1·3
Lead	4·6
Indium	3·5

The variation here is 260 per cent.

* Chem. Soc. Trans. 1890. The values selected refer to solutions of the same strength as those used by Raoult. The results with calcium nitrate and chloride are as yet unpublished.

† Most of these values are but approximations.

‡ Chem. Soc. Trans. 1889, p. 667. The values refer to solutions containing as near as possible one atom of the foreign metal to 100 Na.

If such variations are not too great to be attributed to an excess of two or three atmospheres' pressure (and I certainly think they are), they are undoubtedly too great to permit of a theory being founded on the assumption that they are constant.

It may be noted also that Heycock and Neville's determinations with indium, gold, and thallium extended to solutions as dilute as the gases from these metals would be (if their molecules are diatomic), and that even with these "ideal" solutions the variation amounted to 60 per cent.; had lithium and cadmium been similarly investigated we should, no doubt, have found as great a variation as that quoted above with the stronger solutions.

Direction of the Deviations with Dissolved Substances which give Abnormal Values.

The abnormally low values (*i. e.* half values) which some dissolved substances give are attributed, as has been mentioned, to the polymerization of their acting molecules, and the abnormally high values which some solvents give are, in the same way, attributed to a polymerization of the solvent molecules. This view is accepted by both the supporters and the opponents of the osmotic pressure theory.

In the case of dissolved substances which give abnormally high values (*i. e.* double values), the supporters of the theory hold that these substances are dissociated into their ions, and in this view their opponents cannot agree with them.

The first point to settle, is whether there are any abnormally high values (given by dissolved substances), and it appears to me that there certainly are none.

In the case of the five organic solvents examined, the substances giving the higher and lower values are in the proportion of 100 to 16, and here there can be (and is) no question as to which are the normal ones; even if in the case of water these proportions were reversed, it would be inconsistent to term the lower values the normal ones; but the proportions are actually not reversed, the substances giving high values being to those which give the lower ones in the proportion of 100 to 76. I fail to see therefore what grounds

there are for saying that there are any abnormally high (double) values at all, or what need there is for calling in such a theory as that of dissociation into ions *.

Indeed, there is a distinct difficulty, if not impossibility, in calling the lower values (1°·03) the normal ones; for the normal value with other liquids is 0°·63, and the excess in the case of water could only be explained by the water molecules being more complex than those of other liquids in the proportion of 1·03 to 0·63; in other words, that the water molecule is $1\frac{1}{2}$ H_2O, and that the atomic theory is wrong.

The theory of dissociation into ions is, for many reasons, inacceptable to most chemists, and its application to the results at present under discussion affords, I think, some of the strongest arguments against its acceptance.

If the higher values in the case of water are due to dissociation, we must conclude that all the stronger acids (*e. g.* hydrochloric, sulphuric, and nitric acids) are nearly entirely dissociated, whereas the weaker ones (*e. g.* hydrocyanic, sulphurous, and boric acids) are not so; and similar conclusions must be drawn respecting the alkalies and salts. In fact, we shall be driven to conclude that the more stable the compound is the more easily is it dissociated, and that too by an agent, water, which has, *ex hypothesi*, no action beyond the diluting effect of its mass. Is it possible to maintain that sulphurous acid *exists* intact in solution, while sulphuric acid, which is formed from it and molecular oxygen, with the evolution of 55,000 cal.†, and which must, therefore, be by so much the stabler body of the two, is entirely dissociated?

* Van't Hoff's statement as to Raoult's position respecting the high and low values of substances in water is certainly remarkable; he says (Phil. Mag. 1888, xxvi. p. 99), "Raoult did not discover the existence of so-called normal [meaning small] molecular depression of freezing-point and lowering of vapour-pressure until he investigated organic compounds; their behaviour is almost without exception regular." As a matter of fact, Raoult investigated organic substances *first* (*Ann. Chim. Phys.* 1883 [5] xxviii. p. 133), and when he *subsequently* (*ibid.* 1884 [6] xi. p. 81) found that inorganic substances gave higher values, he did not hesitate to call these latter the normal ones.

If we reverse the application of the word normal, we shall have to admit that of the 150 substances examined in water, not one gave an abnormally low value. A most improbable admission.

† If atomic oxygen were taken, as it should be, the heat evolved would be much greater.

Is it possible to maintain that hydrochloric acid is entirely dissociated at a pressure of three or four atmospheres when we know as a fact that it can exist undissociated at a pressure of only one atmosphere * ?

Indeed, it appears to me that if we acknowledge such a dissociation we must deny the principle of the conservation of energy. Thus, to take one instance out of many, the molecules of hydrogen and chlorine must † be formed from their constituent atoms with a considerable development of heat, say $+x$ and $+y$ cal. respectively, and they react with each other to form 2HCl with a further development of 44,000 cal. ; if this 2HCl, when dissolved in water, be entirely dissociated into its atoms, it must *absorb* $44{,}000 + x + y$ cal., whereas, as a matter of fact, it *evolves* 34,630 cal. : hence there has been a creation of $(78{,}630 + x + y)$ cal. out of nothing, and that, too, through the agency of the water, which is assumed to have no action at all.

Influence of the Nature of the Solvent.

It is found that the nature of the solvent, instead of being without influence on the results, as it should be according to the osmotic-pressure theory, is one of the main factors in determining them.

Thus, for instance, sulphuric and hydrochloric acids give the normal high values when dissolved in water, but solutions of the *same* strength in acetic acid show the abnormal half-values : formic and benzoic acids, methyl, ethyl, butyl, and amyl alcohols, as well as phenol, all give normal depressions with acetic acid, but abnormal ones with benzene. Many other instances of a similar character may be selected from Raoult's data, but none of them show the influence of the nature of the solvent in such a forcible manner as the following :—

* In acetic acid hydrochloric acid is, according even to the supporters of the theory, not only not dissociated into its ions, but even not dissociated into its fundamental molecules, it being present as 2HCl.

† I think that we may say "must." If they were formed with absorption of heat it would mean that the atoms repel each other, and such atoms would, therefore, never combine. When we call a molecule endothermic, we merely mean that it is formed from its constituent atoms with a smaller development of heat than some other compound of the same atoms, not that it is formed from these atoms with an *absorption* of heat.

	Molec. Depress.
H_2O dissolved in 100 H_2SO_4	1·072
„ „ 100 ($H_2SO_4H_2O$) . . .	0·02
„ „ 100 ($H_2SO_4 4H_2O$) . . .	0·08
„ „ 100 ($CaCl_2 6H_2O$) . . .	0·003
„ „ 100 ($Ca(NO_3)_2 4H_2O$) . .	0·007
„ „ 100 (CH_3COOH) . . .	0·55

The variation of this constant (?) calculated on the smallest value amounts to 35,600 per cent. Again,

H_2SO_4 dissolved in 100 H_2O	2·15
„ „ 100 ($H_2SO_4H_2O$) . . .	0·01
„ „ 100 ($H_2SO_4 4H_2O$) . .	0·04
„ „ 100 (CH_3COOH) . .	0·31

Extreme variation 21,400 per cent. Similarly,

$CaCl_2$ dissolved in 100 H_2O	2·773
„ „ 100 ($CaCl_2 6H_2O$) . . .	0·010

Variation 27,600 per cent.; and

$Ca(NO_3)_2$ dissolved in 100 H_2O	2·50
„ „ 100 $Ca(NO_3)_2 4H_2O$.	0·015

Variation 16,600 per cent.

Such numbers can certainly not be accepted as constants, and the values are certainly largely dependent on the nature of the solvent*.

In the present instance the variation cannot always be taken as a proof of the incorrectness of the osmotic-pressure theory, for it may be perfectly legitimate to attribute it sometimes to the various complexity of the molecules of the solvent. But we cannot, however, do so in all cases, as for instance that of formic &c. acids in benzene and acetic acid, where the two solvents have been proved to act normally with the majority of dissolved substances. Yet, in any case, the absence of constancy means the absence of what would have been a strong argument in favour of the theory, and this absence must be felt all the more because there appears to be no simple numerical relationship between the very different values given above.

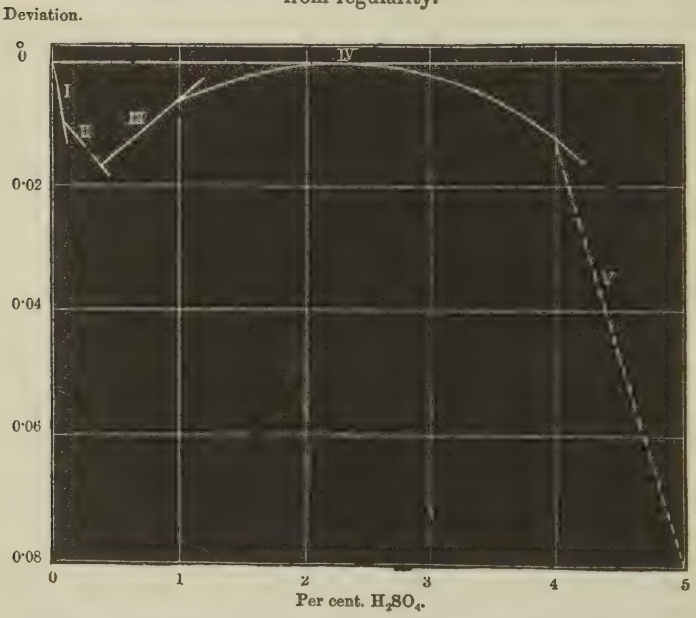
* Combining the variations due to different solvents with those due to different dissolved substances, we get a variation of 153,000 per cent. (4°·6 to 0°·003)

Influence of Proportions taken with very Weak Solutions.

My own recently published results (Chem. Soc. Trans. 1890, p. 331) with sulphuric-acid solutions form, I believe, the only series of determinations as yet made which would, from their number and precision, be at all likely to settle whether the molecular depression is really constant or not with very weak solutions; and the results here can leave little doubt but that it is not so.

Between 0 and 9 per cent. solutions the results are represented by what would with rough determinations appear to be a single line of typical straightness. The deviations from straightness in regions below 4 per cent. (about 0·8 H_2SO_4 to 100 H_2O) are less than 0°·02, and would, therefore, not have been revealed by determinations such as have been made by other investigators. From 4 per cent. to 0 the determinations (each being the mean of several observations) which I made

Deviation of the Freezing-points of weak solutions of Sulphuric Acid from regularity.



show mean error of 0°·0008 at most, and the number of points determined is about 80; and the results obtained leave no

doubt but that the apparently straight line is really composed of four distinct lines, three of which are straight, meeting at (generally) well marked angles at the percentages of 1·0, 0·35, and 0·07 respectively.

The relative magnitude of these changes is best illustrated by drawing a straight line from the freezing-point of water to that of a 2·5 per cent. solution (H_2SO_4 to 200 H_2O ; this was selected merely because at this percentage the diagram attains the highest relative point) and plotting out the deviations of the observed freezing-points from this line. The accompanying figure illustrates the result. The deviation from straightness amounts to at least 20 times the mean experimental error, and 28 per cent. of the total depression. Even if we draw our straight line from a solution of 0·44 (·08 H_2SO_4 to 100 H_2O) per cent. strength—in which the sulphuric acid would be as much expanded as it would when a gas at ordinary pressures—the aberration, at 0·7 per cent., amounts to 0°·008, or 20 per cent. of the total depression.

The molecular depression calculated from these results, together with others obtained with stronger solutions, are given in the following table:—

Depression of the Freezing-point of Water produced by the addition of Sulphuric Acid.

Per cent. H_2SO_4.	Mols. H_2SO_4 to 100 H_2O.	F.-P.	Molecular depression.
0·068	0·0125	− 0°·0369	2°·95*
0·3618	0·0667	− 0·1580	2·37
0·44	0·0811	− 0·1874	2·31
1·057	0·1961	− 0·4214	2·15
2·500	0·4708	− 0·9825	2·09
4·021	0·7692	− 1·600	2·08
6·000	1·1855	− 2·520	2·15
8·585	1·724	− 3·456	2·00
18·492	4·167	−11·83	2·84
29·526	7·692	−34·00	3·74
37·701	11·111	−72·63	6·54

* With still weaker solutions the molecular depression is somewhat greater: with a 0·005 per cent. solution (·0009 H_2SO_4 to 100H_2O) the value found was 3°·4, but the total depression here is so small that I have not included it or similar values in the table.

The depression with solutions weaker than H_2SO_4 to 100 H_2O varies between 2°·08 and 2°·95, 42 per cent., and even between 0·44 and 0 per cent. it varies from 2°·31 to 2°·95, a variation of 28 per cent., numbers which can give no grounds whatever for the statement that the depression, even with these very weak solutions, is independent of the strength.

The only other determinations which, so far as I know, exist with solutions containing less than one dissolved molecule to 100 of the solvent, are those of Heycock and Neville (*loc. sup. cit.*) with certain alloys. Selections from those series which were most fully investigated supply the following data :—

Solvent.	Dissolved substance.	Molecular depression.
100 Na	0·114 Au	5·45°
„	0·296 „	4·74
„	0·966 „	4·54
„	0·097 Cd	3·92
„	0·391 „	3·71
„	0·648 „	3·17*

Direction of the Deviations from Constancy with Stronger Solutions.

According to the theory of osmotic pressure the proportionality between pressure (lowering of freezing-point) and amount of substance dissolved will hold good, as has been remarked, only for very weak solutions. With stronger solutions the dissolved molecules will be brought within the sphere of each other's attraction and will, therefore, exert an abnormally small pressure ; that is, they will lower the freezing-point of the solvent to an abnormally small extent †.

* With thallium, mercury, sodium, and indium, the depression was more constant, but the data were less complete.

† The same will hold good if we attribute dissolution to the balancing of the physical attractions of (1) solvent for solvent molecules, (2) dissolved for dissolved molecules, and (3) solvent for dissolved molecules. The stronger the solution the greater will be (2), and, consequently, the less will (1) be counterbalanced.

But in every case at present known exactly the reverse is the case. Stronger solutions exhibit an abnormally *large* depression, and the freezing-points fall at an increasing instead of a diminishing rate. The table given above illustrates to what extent this takes place in the case of solutions of sulphuric acid in water, the molecular depression increasing from 2°·08 to 6°·54 (213 per cent.), and the examination of six other curves from the sulphuric-acid series, and two of a series with calcium nitrate, supplies us with eight other instances of a similar character. The results may be summarized thus :—

Solvent.	Dissolved substance.	Molec. depression.	Increase.
100 H_2SO_4	1 to 33 H_2O	1°·072 to 1°·253	17 per cent.
100 H_2SO_4	1 to 5·8 SO_3	0·422 to 0·634	50 ,,
100 ($H_2SO_4H_2O$)	1 to 50 H_2O	0·02 to 0·31	1,450 ,,
100 ($H_2SO_4H_2O$)	1 to 50 H_2SO_4	0·01 to 0·197	1,870 ,,
100 ($H_2SO_4H_2O$)	1 to 400 H_2O*	0·078 to 0·108	39 ,,
100 ($H_2SO_44H_2O$)	5 to 60 H_2SO_4†	0·27 to 0·42	55 ,,
100($Ca(NO_3)_24H_2O$)	1 to 150 H_2O	0·007 to 0·326	45,600 ,,
100($Ca(NO_3)_44H_2O$)	1 to 33$Ca(NO_3)_2$	0·015 to 0·179	11,000 ,,

It is true that these results have only recently been placed at the disposal of the supporters of the osmotic-pressure theory, but it is otherwise with the following values in the case of alcohol (Raoult, *Compt. Rend.* xc. p. 863) and calcium chloride (Hammerl, *Wien. Sitzungsber.* lxxviii. p. 59)‡:—

Solvent.	Dissolved substance.	Molec. depression.	Increase.
100 H_2O	1 to 7 $CaCl_2$	2°·773 to 6°·9	150 per cent.
100 ($CaCl_26H_2O$)	1 to 800 H_2O	0·003 to 0·099	3200 ,,
100 H_2O	1 to 13 C_2H_6O	1·11 to 1·18	6 ,,

The increase in the case of alcohol is small, but still there

* With 60 H_2O the depression is 0°·067, so that it first decreases and then increases.

† With 1 mol. H_2SO_4 it is 0°·40.

‡ In each case I give the values as determined from my own results, but these are in good accord with those referred to above.

is an increase, and not a decrease ; while in the case of water in the hydrate of calcium chloride the increase is almost the greatest at present known.

Irregularity of the Deviations with Weak Solutions.

That the deviations from constancy are not regular, and are not even always in the same direction, will be seen on referring to the results with sulphuric acid dissolved in water (p. 363) ; the depression in the freezing-point of the tetrahydrate, both by water and sulphuric acid, supplying two other instances of a similar character.

Since every fact which can be used to test whether the theory of osmotic pressure is a true explanation of the nature and behaviour of solutions, either fails to give any evidence in its favour, or else gives evidence directly opposed to it—evidence, often, of the strongest possible character—this theory can certainly not yet be regarded as established.

XLVIII. Dr. S. P. Thompson described "*Bertrand's Refractometer*," and exhibited the capabilities of the instrument before the Society*.

(Abstract.)

Its action depends on total reflexion. The refractometer consists of an hemisphere of glass about 8 millim. diameter set at the end of a tube, the plane face being outwards and inclined at about 30° with the axis. One side of the convex surface of the hemisphere is illuminated through a piece of ground glass set about perpendicular to the plane face. The hemisphere is viewed through an eyepiece focused on a scale divided to tenths of millimetres, placed within the tube. The instrument is particularly useful for mineralogical specimens and liquids. The procedure in the latter case is to smear a film of the liquid over the plane face of the hemisphere, and, by looking through the eyepiece, determine the scale-reading

* Read March 7, 1890.

of the line which separates the light and darker portions of the field. A reference to a calibration-table gives the refractive index.

In experimenting with solids, a thin film of very dense liquid (supplied with the instrument) is placed between the specimen and the glass, and the procedure is then as above. The refractive index of opaque solids can be determined in this way. In using the instrument for minerals, great care must be taken not to scratch the glass. The handiness of the refractometer and its perfect portability (its dimensions being about 5 centim. long by $2\frac{1}{2}$ centim. diameter) are great recommendations.

Mr. Blakesley asked to what accuracy the scale could be read, and whether the sensitiveness of the instrument was at all comparable with that of other methods.

Prof. Dunstan inquired if it could be used with volatile liquids.

In reply, Dr. Thompson said that with non-homogeneous light the scale could be read to one division, but with a sodium-flame one tenth of a division could be estimated. For volatile liquids a drop may be used instead of a film, or the evaporation of a thick film may be retarded by a cover-glass.

XLIX. *The* Villari *Critical Points of Nickel and Iron**. *By* Herbert Tomlinson, *F.R.S.*†

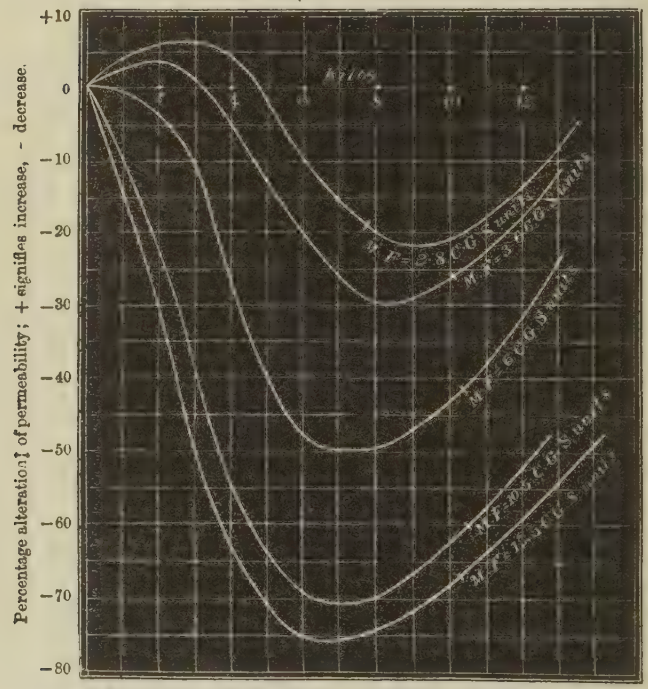
We owe to Villari the discovery that the magnetic permeability of iron is increased by longitudinal traction provided that the magnetizing force does not exceed a certain limit, but beyond this limit the traction produces decrease of permeability so that for a certain value of magnetizing force, known as the Villari Critical Point, longitudinal traction has no effect in either direction. Several experimenters have

* Read March 21, 1890.

† The author begs to acknowledge with thanks the assistance which he has received in this investigation from the Elizabeth Thompson Science Fund, U.S.A.

verified and extended Villari's discovery, notably Sir William Thomson and Prof. Ewing, but, as far as the author is aware, no observer has been as yet able to find a similar critical point for nickel. If, however, we confine ourselves to the consideration of the *temporary* magnetization*, a Villari critical point in nickel can be detected with comparative ease. In the present investigation the ballistic method of observation has been employed, the arrangements of which have been already described†. The nickel wire used contains nearly 98 per cent. of nickel and only 0·7 per cent. of iron; the iron wire

Fig. 1.—Iron.



* The magnetization which disappears on the removal of the magnetizing force.

† Phil. Mag. xxv. p. 372.

‡ If P_0 and P_n represent respectively the permeability with no load and a load of n kilos, the percentage alteration of permeability is taken to be $\frac{100(P_0-P_n)}{P_0}$ or $\frac{100(P_n-P_0)}{P_0}$ according as the permeability is decreased or increased by loading.

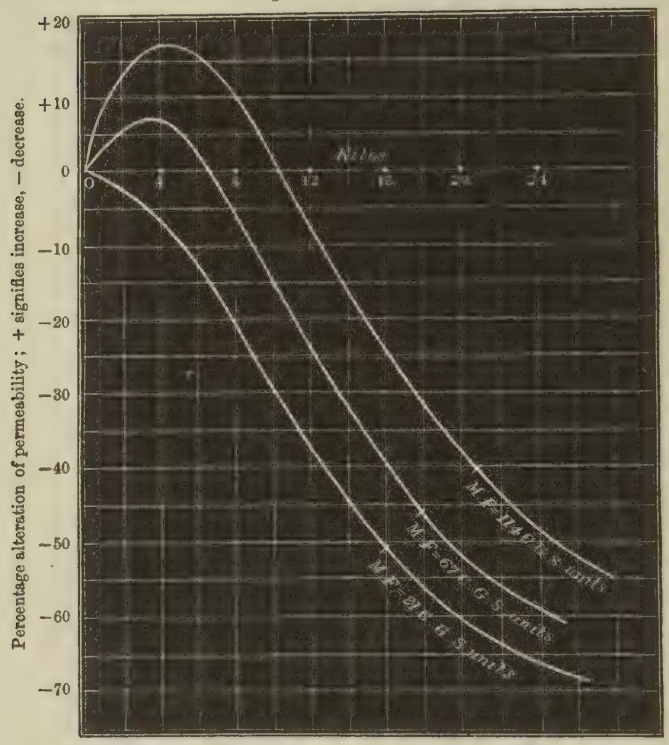
is also, judging from its small specific resistance, nearly pure. The mode of conducting the experiments may be illustrated by the aid of fig. 1; this figure refers to a piece of well-annealed iron wire 1 millim. in diameter. The wire was in the first instance loaded with 12 kilos (the highest load employed in the experiment), and was subjected to a magnetizing force of 15·5 C.G.S. units which was alternately applied (always in the same direction) and removed until the deflexions of the ballistic galvanometer became constant. The load was then entirely removed, and the same force was again applied and removed as before. A load of 2 kilos was now put on the wire, and the same operations were repeated with this and the subsequent loads up to 12 kilos. Next the load was again entirely removed, and a similar series of observations was made with a magnetizing force of 10·5 C.G.S. units, and so on until the force had diminished to 2·8. The entire set of experiments was then repeated with ascending forces up to 15·5; this set did not sensibly differ from the one taken with descending magnetizing forces. As might be expected, each change of load produced an alteration in the *residual* magnetization, but this alteration does not concern us, and only the *temporary* changes, *i.e.* those which take place after repeated application of the force and when the deflexion on closing the battery-circuit is equal to that on opening it, are recorded in the figure.

It may be noticed that the Villari critical point depends upon the load, the greater the load the smaller the critical value of the force. This fact has been already observed by Sir William Thomson, but for a given load the Villari critical point is *much lower* when only the *temporary* magnetization is taken into account than when, as in Sir W. Thomson's experiments, the *whole* magnetization is observed. It occurred therefore to the author that though it might not be possible to reach the critical force for nickel as far as the total magnetization is concerned, such a value could nevertheless be found for *temporary* magnetization, and this has proved to be the case.

Fig. 2 refers to a nickel wire of ·8 millim. diameter, which shows Villari critical points quite as striking as those of iron, but with this difference, that the critical value is *greater the*

greater the load, and for a given load is much greater in nickel than in iron.

Fig. 2.—Nickel.

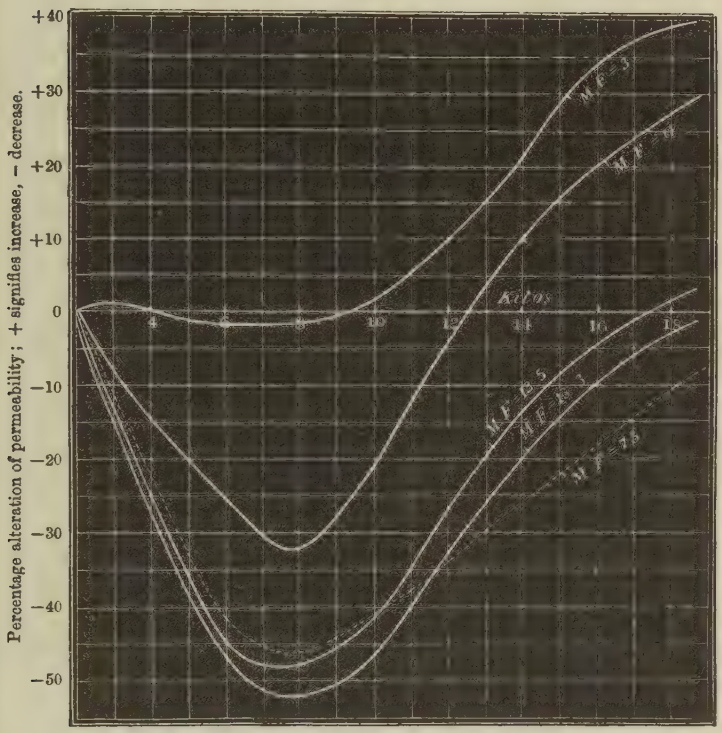


With iron wire the curves may, and do, sometimes cut the load line *twice*, so that for a given value of magnetizing force there may be *two* loads which have no effect on the temporary magnetization. In fig. 1 none of the curves *actually* cut the load-line a second time, because the loading was not carried quite far enough; but with subsequent experiments made with the same wire and with the same loads at a temperature of 200° C. the two points of cutting were easily obtained, within certain limits of magnetizing force, whilst the next figure shows that with greater loads the second point can be reached at the ordinary temperature of the room.

Fig. 3 illustrates the effect of loading on an iron wire of ·077 millim. diameter, which in the first instance was

well annealed, but was subsequently loaded to very nearly its breaking-point, and was in consequence permanently

Fig. 3.—Iron*.



lengthened by about 10 per cent., whereas the wire referred to in fig. 1 was not loaded sufficiently to produce any sensible permanent extension. For a magnetizing force of 3 C.G.S. units the curve cuts the load-line twice, but for higher forces the first cutting-point disappears and we have only the second one.

The maximum decrease of permeability is greater the greater the magnetizing force up to a certain limit of the latter, which limit other experiments (not illustrated by the figure) proved to be about 16 C.G.S. units. Beyond this limit the maximum decrease begins to become slowly less, as

* In this experiment a load of 2 kilos was left permanently on the wire, so that the initial load is 2 instead of 0.

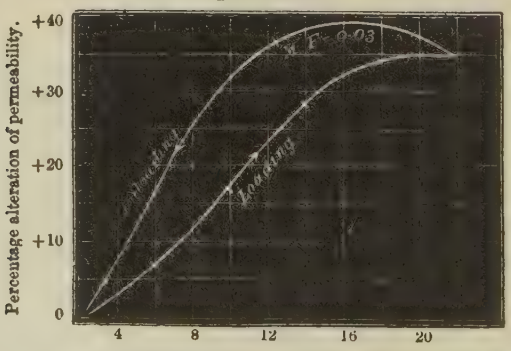
is shown by the dotted curve of the figure for which the magnetizing force was 73.

The curves for nickel can never be made to cut the load-line twice, for the wire of fig. 2 was stretched within one or two per cent. of its breaking-point, suffering thereby a permanent elongation of about five per cent. of its original length. The curves, however, show a *maximum* decrease of permeability with a load of about 24 kilos.

The observations hitherto recorded were only made at the different stages of *loading*, but subsequent ones showed that with nickel the readings on *unloading* were very nearly the same as those on *loading*, there being very little lagging of mechanical strain behind stress. Prof. Ewing has pointed out also that nickel exhibits very little *magnetic* hysteresis. With iron there is a very sensible lagging of mechanical strain behind stress, but the only effect of taking a mean between the observed changes on loading and unloading would have been to make the curves more parabolic in shape and not to alter their main character.

For the next two figures observations were made both on loading and unloading, and though the magnetizing force employed was here very minute, the curves show pretty nearly the same amount of lagging of mechanical strain behind stress which is experienced when much higher forces are used. For these very minute forces, of course, the apparatus had to be made much more sensitive, and the

Fig. 4.—Iron.

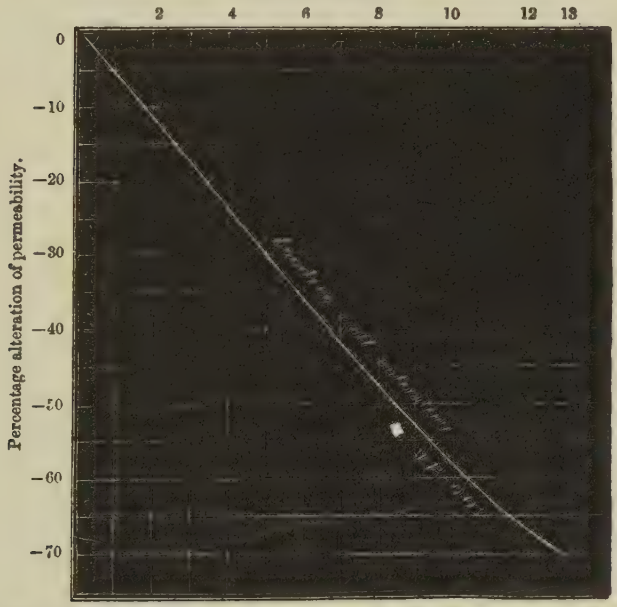


number of turns in the secondary coils was very great. Fig. 4 refers to the same kind of wire as that used in the first

experiment. The wire in this case was, however, previously loaded to nearly its breaking-point. If the mean be taken of the percentage alterations of permeability on loading and unloading, the curve becomes exactly parabolic, with its axis vertical and its vertex at the point where the vertical from the load 16 kilos. cuts the curve. Both the loading and unloading curves lie entirely *above* the load-line, and there is no point of cutting this line.

Fig. 5 deals with a nickel wire of the same degree of purity as the first but 1½ millim. in diameter. The stress

Fig. 5.—Nickel.



1 kilo on the lever represents a stress of 2·68 kilos per sq. millim.

was carried by the aid of a lever to very nearly the breaking-point of the wire. There is in this case no perceptible lagging of mechanical strain behind stress, the curve on loading being identical with that on unloading. With nickel the curve lies entirely *below* the load-line and never cuts the line.

In figures 4 and 5 the curves are described as having been obtained with a magnetizing force of 0·03 C.G.S. units; but other experiments showed almost precisely the same

curves to be obtained for any force between 0·03 and 0·3 C.G.S. units. Between these limits of force the *whole* of the magnetization is practically *temporary* provided the wires have been stretched to the extent these were.

From what has been said and from the figures it is easy to see what would be the general character of the curves, showing the relation between change of temporary permeability and change of load for any magnetizing force whatever. If a piece of iron wire be tested with loads nearly up to the breaking-point, then for magnetizing forces ranging from 0 to somewhere about ·3 C.G.S. units the curves will be *identical* with those of fig. 4, and will not cut the load-line. For a value of force above ·3 the curves will cease to be identical, the maximum increase of permeability will become less and less as the force increases until for a certain force the curve begins to cut the load-line. As the force increases beyond this value the point of cutting approaches nearer and nearer to the origin of coordinates, and the curve begins eventually to cut the load-line in two points. The first point of cutting, as the force increases still further, gets nearer and nearer to the origin, whilst the second point gets further and further away from it; and when the magnetizing force reaches a value but little over 3 C.G.S. units, the first point vanishes and the second point only remains. Finally, with a sufficiently high force it would seem that the second point also cannot be reached and the curve now lies *entirely beneath* the load-line, whereas for very minute forces it lies *entirely above* this line.

With nickel the curves for very minute magnetizing forces are, like those of iron, exactly the same for different values of the force, but instead of lying *above* the load-line lie entirely *below* it. Beyond a certain limit of the force the curves cease to be identical and the maximum decrease of permeability begins to become less and the curve near its origin to bulge more and more towards the load-line, until for a value of force a little over 21 C.G.S. units the permeability begins to increase with small loads and to cut the load-line in one point, which gets shifted further and further away from the origin as the force increases.

Mr. Shelford Bidwell said that Prof. J. J. Thomson,

reasoning from the change of length by magnetization, had predicted a Villari point in cobalt when compressed, and this was verified experimentally. On applying similar reasoning to nickel, he (the speaker) did not expect to find a Villari point, and both Sir W. Thomson and Prof. Ewing had searched in vain for one. In some experiments, not yet completed, he had examined the behaviour of nickel, both loaded and unloaded, when subjected to various magnetizing forces. These show that the metal always contracts when magnetized. For no load the contraction at first increases with the magnetizing force, but attains a maximum. With a moderate load the contraction is less for small forces, but for larger forces becomes equal, and then exceeds the contraction of the unloaded wire. For greater loads the contraction is less than when unloaded for all values of the force.

L. *On* Bertrand's *Idiocyclophanous Prism.* *By* Prof. S. P. Thompson, *D.Sc.**

(Abstract.)

This hitherto undescribed prism is a total-reflexion one, made of calc-spar, which shows to the unaided eye the rings and crosses such as are seen when a slice of spar is examined by convergent light in a polariscope. The space is cut so that the light after the first reflexion passes along the optic axis, and after the second reflexion emerges parallel to the incident light. The rings and brushes are present in pairs, but two pairs may be seen by tilting the prism to one side or the other. This was demonstrated before the Society.

Prof. Thompson also exhibited a similar prism cut from quartz. Owing to the feeble double-refracting of the substance no conspicuous rings could be seen, but when examined by the lantern traces of such rings were visible.

* Read March 21, 1890.

LI. *On the Shape of Movable Coils used in Electrical Measuring-Instruments.* *By* T. Mather, *Assistant in the Physical Department, Central Institution**.

It is with some diffidence that I venture to bring the subject of this note before the Society, because it concerns such a fundamental point in the construction of measuring-instruments having movable coils, such as d'Arsonval galvanometers, electro-dynamometers, wattmeters, &c., that it is almost certain to have been worked out before. However, as the matter is not touched upon in the ordinary text-books, I bring it forward in the hope that it may serve to recall attention to a subject which, judging from the construction of such instruments as are in common use, has been almost entirely ignored.

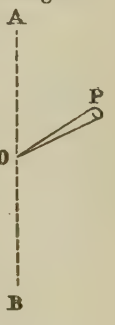
In a paper "On Galvanometers," by Prof. Ayrton, Dr. Sumpner, and the writer, read before the Society on January 17th, it was pointed out that the coils of d'Arsonval galvanometers should be narrow and long, and that there should be no internal core. Mr. C. V. Boys had evidently arrived at the same conclusion long before the date referred to, for the coil of his radio-micrometer is a proof of this.

The object of this paper is to determine the best shape of the section of the coil perpendicular to the axis about which it turns.

The subject will be dealt with as concerning coils suspended in uniform fields, but similar reasoning may be applied to other instruments in which movable coils are used.

Let the point O (fig. 1) represent in plan the axis about which the coil turns, and assume it is placed in a magnetic field whose direction is perpendicular to A B.

Fig. 1.



Let P be an element of the section of the coil, then the deflecting moment exerted by unit length measured at right angles to the paper is

$$HCar\sin\theta\,; \quad . \quad . \quad . \quad . \quad (1)$$

where H is the strength of field, C the current-density per unit area, a the area of the element, and r its distance from the axis.

* Read March 21, 1890.

The moment of inertia of the element about O will be

$$w a r^2, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

where w is the mass per unit cube.

Now in ordinary commercial instruments it is important that the period of oscillation should not be inconveniently long, and that the power consumed by the instrument should be as small as possible ; then, since for a constant period the controlling moment at unit angle must be proportional to the moment of inertia, the problem resolves itself into finding the shape of the section such that the total deflecting moment for a given total moment of inertia is a maximum.

The ratio of the Deflecting Moment to the Moment of Inertia for the element above considered is

$$\frac{\mathrm{HC}ar \sin\theta}{w a r^2}, \quad i.\ e. \quad :: \frac{\sin\theta}{r},$$

since H, C, and w may be considered constants.

The ratio $\frac{\sin\theta}{r}$ thus is a measure of the efficacy of the element and its position.

If we now draw a curve whose polar equation is

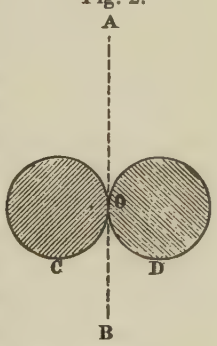
$$r = x_1 \sin\theta,$$

then, as in Maxwell (vol. ii. p. 332), it may be shown that a given length of wire wound within this space is more efficient than if wound outside it. Hence the curve $r = x_1 \sin\theta$, *i. e.* a circle tangential to A B at O, is the best form of the section of a movable coil, just as in the case of sensitive galvanometers the best shape is given by

$$r^2 = x_1^2 \sin\theta.$$

The complete section is given in fig. 2, and consists of two circles touching at O. The circles C and D have section-lines in different directions to indicate that the current passes in opposite directions in the two.

Fig. 2.



The problem may be treated in another way ; for it resolves itself into finding the shape and position of an area having a given moment of inertia about a point in its plane such that the moment of the area about a coplanar line through the point is a maximum.

Taking the point as pole, and the line as the line of reference, the expression

$$\iint r^2 \sin\theta \, dr \, d\theta \text{ is to be a maximum,}$$

whilst

$$\iint r^3 \, dr \, d\theta \text{ is constant.}$$

This may be solved by the Calculus of Variations with the result above obtained. By this method Dr. Sumpner (to whom the writer suggested the problem) arrived at the solution some time before the author considered the subject from the first point of view.

In order to illustrate the sort of improvement obtained by winding coils to this shape, a table is appended, the first column of which shows various shapes of section, the second column the value of the ratio moment of area ÷ moment of inertia of area, and the fourth column the deflecting moments of the various shapes all having the unit moment of inertia. From this last column it may be seen that the ordinary Siemens dynamometer type of coil, and those of d'Arsonval's galvanometers and wattmeters, are far from being the most efficacious.

The numbers in column 4 are obtained on the assumption that the coils are long in proportion to their breadth; and when this condition is not fulfilled (as it very seldom is), the numbers would be still further reduced, for the parts of the wire at right angles to the axis of suspension add to the moment of inertia but not much to the deflecting force.

This consideration is of importance where the coils are of sections such as (4), (7), (8), (9); for as the coils are wide, the useless moment of inertia sometimes amounts to one fifth the useful. In such a case as (8) the real numbers representing the efficacy of the coil, as compared with one made to section (1) of the same moment of inertia, would be about 1 to 3. These latter numbers represent approximately the efficacy of the coil of an ordinary Siemens dynamometer reading up to 60 amperes, as compared with that of a coil made to the best shape. It is thus possible to increase the deflecting moment for a given current threefold by changing the shape of the movable coil and arranging the fixed coils to produce the same strength of field as at present used.

Of course the new coil would be heavier than the old one, and would therefore require a stronger suspension which might introduce more vagueness of zero; but as the friction at the mercury-cups is considerable, the necessity for using a stronger suspension would not seriously interfere with the accuracy of the instrument.

It will be noted that in the preceding part of this paper the moment of inertia of the suspended coil has been assumed constant; it is perhaps desirable to give reasons why this assumption is made.

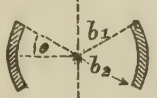
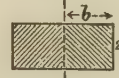
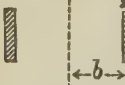
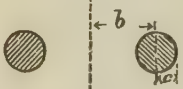
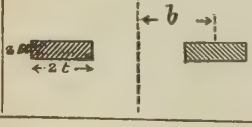
In most instruments having movable coils which are in ordinary use, the current is led into and from the coil either by mercury-cups, or by wires which also serve as the control, or by flexible wires independent of the control.

Taking the first case of mercury-cups, it may be noted that, owing to friction and viscosity, a certain minimum control is necessary to give the requisite definiteness of zero. As time is generally of importance in making commercial measurements, the period of oscillation may be taken as constant. Therefore, as both control and period are determined by circumstances other than those affecting the shape of the coil, the moment of inertia may be taken as constant.

When the current is led into the coil by the torsion-wires, the smallest size of the latter is usually determined by considerations of heating, and their length is limited by questions of portability, compactness, and resistance. This means that the control cannot be diminished below a certain minimum, and hence, by reasoning as in the last paragraph, the moment of inertia of the coil must not exceed a certain value.

The third case, where the leading-in wires are independent of the control, the conditions are somewhat similar to those in the first case; for the size of these wires is determined by considerations of heating by the current to be measured, and they necessarily possess some viscosity or internal friction, which requires a certain control to give definiteness of zero.

In addition to the assumption of constant Moment of Inertia, the field in which the coil swings is taken to be uniform and parallel; and consequently the reasoning will not apply to radial fields which exist in instruments such as the new d'Arsonval milliamperemeter, having a range of 180°.

	Shape of Section.	Deflecting moment / Moment of inertia	Proportions in particular cases.	Deflecting moment per unit Moment of inertia.	Instrume
1.		$\frac{4}{3b}$.	.....	1·02	
2.		$\frac{4\sin\theta}{3b\theta}$.	$\theta=30°$. $\theta=45°$.	0·91 0·95	
3.		$\frac{8}{3\pi b}$.	.....	0·80	
4.		$\frac{4\sin\theta}{3\theta}\frac{b_1^3-b_2^3}{b_1^4-b_2^4}$.	$\theta=19°$, $b_2=\frac{12}{13}b_1$	0·44	Low-resista d'Arsonv
5.		$\frac{3b}{2(b^2+a^2)}$	$2a=b$. $4a=b$.	0·97 0·91	
6.		$\frac{3}{4b}$.	.....	0·79	
7.		$\frac{3b}{t^2+a^2+3b^2}$.	$a=\frac{1}{5}b$, $t=\frac{1}{15}b$ $a=\frac{1}{4\cdot8}b$, $t=\frac{1}{26}b$	0·47 0·38	Jolin's High-resista d'Arsonva Wattmeter (and Co.), amperes.
8.		$\frac{b}{b^2+\frac{c^2}{2}}$	$c=\frac{1}{11}b$.	0·40	Dynamomet (Siemens) amperes.
9.		$\frac{3b}{t^2+a^2+3b^2}$	$a=\frac{1}{19}b$, $t=\frac{1}{2\cdot5}b$ $a=\frac{1}{22}b$, $t=\frac{1}{2\cdot7}b$	0·53 0·49	Dynamomet (Siemens) amperes. Wattmeter (Siemens), amperes.

In such case it would be advantageous to make the coil of a section similar to that in No. 4 (see table), keeping the radius b as small as possible.

The shape given in No. 1 of table cannot be imitated exactly in practice, for insulation-space is required between the two halves, and unless the coil is used in a zero-instrument, parts near the axis might oppose the rest when a considerable deflexion exists. Shapes (3), (5), and (6) are also open to the same objection, but this is easily surmounted by allowing a small space to exist about the axis; in fact some such space is almost necessary to enable the coil to be wound conveniently.

Again, in Siemens dynamometers for fairly large currents it would be difficult to make the coil of the best shape, owing to the space required for the mercury-cups being considerable; but a much closer approximation to this shape than the one often employed could be used.

In ordinary d'Arsonvals, which deflect through a considerable angle, the shape should be modified so that no part of it crosses the line A B (fig. 2) even at the maximum deflexion, for any such part would oppose the rest of the coil. The resulting figure would be lemniscate-shaped. There are some cases, however, such as deflexional dynamometers, where it is advisable to make the coil of such a shape as to cross the line A B when deflected; for then the deflexion would increase less rapidly than the square of the current, and hence the instrument would have a greater useful range for a given length of scale.

Mr. C. V. Boys said he had, when working at his radio-micrometer, arrived at a shape similar to that recommended in the paper. He also noticed a peculiar relation true for all shapes where the length parallel to the axis of rotation is great compared with the breadth. Suppose a coil of any dimensions, then another coil of half the breadth and double the length and cross-section will be dynamically, electrically, and magnetically the same as the original; for the moment of inertia, the electric resistance, and the enclosed magnetic field are equal. The above relation is also true where the breadth is not small if the cross-pieces be thickened near the axis, so as to make their moment of inertia proportional to their

length. He inquired whether the author had considered the subject of grading the movable coils; he himself was of opinion that, unlike fixed galvanometer-coils, the wire near the axis should be thicker than that further away.

The President remarked that, in 1881, Prof. Perry and himself exhibited a Wattmeter at the Society of Arts whose movable coil somewhat resembled one of those in the paper, which gave a deflecting moment of 95 per cent. of the maximum. In designing the instrument they had felt that the ordinary method of using a comparatively large swinging coil was not the best, and this led them to the shape adopted.

LII. *Considerations on Permanent Magnetism.*

By M. F. Osmond*.

Iron possesses at least two molecular states, α and β. α is soft iron, that is iron which has been annealed and cooled slowly. It changes into β during heating, at a certain critical temperature, and in part preserves this state during cooling, the more so the quicker the cooling and the larger the proportion of carbon, manganese, and tungsten that it contains. Some β iron is also produced during cold hammering. Iron deposited electrically is also of the β variety.

A bar of steel, allowance being made for the carburets and other compounds simply in mixture, can then be looked upon as an intimate mixture of α iron and β iron in relative proportions, which can change from point to point but are determinate at any point. That being so, let us consider β as forming in a steel bar a porous framework, not changing under the influence of currents and of magnets. On the other hand, let us look upon α as composed of particles polarizable under such influences. (The actual mobility of such particles, admitted by Hughes, appears from the sounds and changes of volume which take place during magnetization.) The particles α so polarized will form so many small elementary magnets, which the magnetizing force has displaced from their former normal position of equilibrium, and

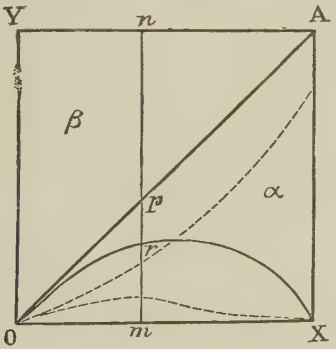
* Read April 18, 1890.

which tend to take up this position again so soon as the external force ceases to act.

But in presence of the rigid network of iron β these polarized particles α are conceived as catching in the pores of the β structure, and immovable in this position, thus resulting in a permanent magnet. The temporary magnetism and the permanent magnetism determined by the action of a certain magnetizing force at a point of a bar will be therefore a function of the relation $\frac{\alpha}{\beta}$ at this point.

As an attempt to represent these ideas graphically, let there be a square, O A X Y, of which the side represents 100 parts. Every line $m\,n$ parallel to the axis of Y meeting in p the diagonal O A represents a steel containing $\frac{mp}{100}$ of iron α, $\frac{np}{100}$ of iron β.

At saturation the temporary magnetism, or rather the sum of the polarized particles, will be exactly equal to the whole of the iron α, *i. e.* to mp. α varying from 0 to $\frac{100}{100}$, the temporary magnetism will be represented by the diagonal OA.



As to the permanent magnetism, in regarding it as the relative number of particles of α which remain polarized after the cessation of the magnetizing force, it will be a fraction of mp, all the greater as $\frac{\alpha}{\beta}$ is smaller for the steel under consideration. In other words, the polarized particles α will be more immovable the closer the network β.

Under the supposition that the quantity mr of the particles α which remain polarized after the interruption of the magnetizing force (that is to say, the permanent magnetism as defined) is proportional to the percentage of iron β, $\frac{np}{100}$, we can write

$$mr = mp \times \mathrm{K}\frac{np}{100},$$

K being a numerical coefficient equal to or less than unity.

Since

$$mp = m\mathrm{O} = x,$$

$$np = 100 - mp = 100 - m\mathrm{O},$$

$$= 100 - x,$$

$$y = x \times \mathrm{K}\frac{100-x}{100},$$

$$= \mathrm{K}\left\{x - \frac{x^2}{100}\right\}.$$

Accordingly the permanent magnetism is represented by a portion of a parabola whose axis is parallel to that of Y. It vanishes for $\alpha=0$, $\beta=0$ (as could be foreseen from the properties of the irons α and β, which have served as the grounds of the argument), and is a maximum for $\alpha=\beta$.

The number of the particles capable of remaining polarized after the interruption of the magnetizing force will at most form a quarter of the whole mass of the iron.

If the magnetizing force be too feeble to produce saturation, the lines OA, OrX will take up a position indicated approximately by the dotted lines.

These considerations agree fairly well with many observed facts.

Thus all causes which tend to open, even temporarily or only in certain directions, the network of the iron β (expansion, shocks, vibrations) set at liberty some of the polarized particles of α, and diminish the permanent magnetism.

The greater the proportion of β iron the more difficult becomes the polarization of α, in the diminished pores, and the greater must the magnetizing force be to produce saturation.

When the proportion of carbon or of manganese is so

great that the whole of the iron on quenching or even without this takes up the state β, a substance will result which is incapable of taking up magnetism ; for example Spiegel-iron containing 25 per cent. of manganese and upwards, and Hadfield's steel. Since hardening takes place more energetically at the surface than in the interior of a bar, the relation $\frac{\alpha}{\beta}$ will always be smaller there. The permanent magnetism remaining after saturation will therefore be stronger at the surface if $\alpha > \beta$, which is the common case. It would be more feeble if $\beta > \alpha$.

Magnets formed of laminæ are in general more powerful than those of the same volume in a single piece, because the laminæ are more hardened. But the contrary should be the case with metals in too hard a condition.

It is difficult after a bar has been magnetized to saturation in one direction to bring it to a moment of the same value in an opposite direction, because certain particles of the α metal being held immovably cannot be displaced, &c.

The iron β can also be looked upon as a medium whose viscosity increases as its percentage in the mixture is greater.

We can compare these ideas with the experiment of Hughes, who has obtained permanent magnets by mixing iron filings and wax and melting the latter, which in setting held fixed the polarized portions of iron.

Mr. Swinburne asked if the theory would account for the increase of induction which occurs when the circuit of a permanent magnet is closed ; most theories founded on the orientation of particles by the magnetizing force seemed defective in this respect. Some time ago he had suggested that the permeability of iron should be tested by first magnetizing it one way, and then at right angles to the first direction ; recently he had been informed that no increase of permeability was observed when the experiment was performed.

Prof. Perry said he had subjected iron to magnetization in one direction, and found the permeability for small forces in a direction at right angles much smaller than he had anticipated; the first magnetizing force was kept constant when the small perpendicular one was applied.

Mr. Swinburne thought that for such small perpendicular forces the permeability should be nearly infinite. He also said there seemed a sort of angular hysteresis in iron; for if a loose-running armature was turned slowly round by hand, it would come back two or three degrees when left free.

The President remarked that, as far as he could see, M. Osmond's theory does not account for the great influence which a small percentage of tungsten has on the magnetic property of steel, and all theories which failed in this particular must necessarily be imperfect.

LIII. Prof. Rücker described the results of *some Recent Magnetic Work* undertaken by himself and Prof. Thorpe in connexion with their Magnetic Survey of the United Kingdom*.

In a paper read before the Royal Society last year they have shown that the United Kingdom can be divided into seven or eight distinct districts, in each of which a source of disturbance seems to exist. The probable cause of these disturbances the authors believe to be the presence of "magnetic rocks." If this be true, an intimate connexion should exist between the magnetic elements of a district and its geological structure. If a magnetic mass be supposed to exist below the surface, then one would expect the vertical force to be greatest above that mass, whilst the horizontal disturbances would tend towards the mass, and would differ in direction on its opposite sides.

With a view to testing more fully than heretofore the magnetic constitution of the districts above referred to, the work (the results of which Prof. Rücker now described) was undertaken.

Two test districts were selected in which numerous observations were taken; one of these is situated on the west coast of Scotland, and the other embraces parts of Lincolnshire and Yorkshire. The first of these districts was chosen for two reasons:—First, because it is very disturbed, and, owing to the vast masses of basalt, would be a specially difficult one; and secondly, because it is near the borders of their survey, and

* Read April 18, 1890.

is thus particularly open to suspicion that the calculated values of the elements may be inaccurate.

By the aid of magnetic maps, Prof. Rücker showed that a centre of disturbance exists to the west of Iona and south of Barra. Both the vertical force and the horizontal disturbances indicate the same position, and hence favour the hypothesis.

In the Lincolnshire and Yorkshire district, extending from the Wash to Appleby, there is a region in which the horizontal disturbances along the East side tend towards the West, whilst along the West side they are, on the whole, directed easterly. This points to a ridge of magnetic material along the middle of the region, and from numerous observations the approximate position of the ridge-line has been determined over a length of 150 miles. In Linconshire the ridge follows closely the line of Wolds, whilst at the Kettlewell end it is directed towards the Cumberland Lake-district. Places of maximum vertical force occur near Market Weighton and Harrogate.

The observers were directed to Market Weighton by Prof. Judd as being situated where the Liassic strata thins out. Harrogate stands on the summit of an anticlinal. The fact that such places of maximum vertical force exist where, according to hypothesis, they would be expected, tends to confirm the supposition, and thus to demonstrate that a very close connexion exists between the magnetic elements and the geological structure of a country. This is further confirmed by the observation that in Lincolnshire the line of disturbance follows closely the line of the Wolds, and that from Market Weighton it passes across the plain of York to the Millstone Grit and Limestone, which are the oldest rocks in the neighbourhood.

Mr. Whipple said he had listened with very great pleasure to the interesting details and results given by Prof. Rücker. Never, to his knowledge, had a magnetic survey been conducted with such minuteness and accuracy. The older surveys related chiefly to declination, whilst the determinations of intensity were made in a very primitive way. He looked forward with interest for other important results, which he felt sure would follow from the admirable survey of Professors Rücker and Thorpe.

LIV. *On the Distribution of Flow in a Strained Elastic Solid.*

By Charles A. Carus-Wilson, *B.A., A.M.I.C.E., Demonstrator in the Mechanical Laboratory at the Royal Indian Engineering College, Coopers Hill*.*

If a metal bar of uniform section throughout its length be subjected to uniform longitudinal stress, the elements of the bar will become distorted ; if the strain should exceed the limit of elasticity the distortion will be partly permanent.

In consequence of the straining of the elements, every element will experience a displacement relative to three fixed axes in the bar ; say $o\,x$ parallel to the length of the bar, $o\,y$ parallel to one side, and $o\,z$ parallel to a second side, both $o\,z$ and $o\,y$ being at right angles to $o\,x$: such displacement is generally spoken of as "flow."

It is clear that if the metal be homogeneous, the stress uniform, and the section at right angles to the axis everywhere the same, an element will experience a displacement at right angles to the planes $x\,o\,z$, $y\,o\,z$ proportional to its distance from these planes ; since such displacement depends on the straining of the elements between it and these planes. Hence a bar with its sides originally parallel to $o\,x$ will remain so after a certain amount of flow has taken place, and lines drawn on the sides respectively at right angles to and parallel to $o\,x$ will remain so, since the displacement parallel to $o\,x$ will be proportional to the distance from the plane $y\,o\,z$.

The question I propose to discuss is how to determine the distribution of flow when the section at right angles to $o\,x$ varies in any way as we pass along $o\,x$.

I will for simplicity consider a plate in which the thickness parallel to $o\,y$ is considerable compared with the thickness parallel to $o\,z$, and I will consider the displacements in the plane $x\,o\,y$.

We may take a plate in which there is a gradual reduction of section, in which there is what is commonly called a "shoulder," as in fig. 2.

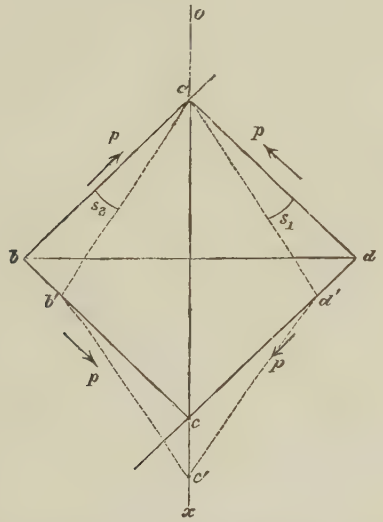
It is required to determine the displacements in the plane of the paper—the plane $x\,o\,y$: in other words, supposing the

* Read May 2, 1890.

plate covered with a series of lines parallel to $o\,x$ and to $o\,y$ respectively, it is required to determine the curvature of these lines due to flow.

The distortion of the elements is caused by the shearing-strain induced in the bar when subjected to longitudinal stress. Since this shearing-strain is a maximum at 45° to the axis $o\,x$ of the bar, it will be convenient to consider the bar divided

Fig. 1.



into a number of small elements with sides making angles of 45° to the axis $o\,x$. Every such element as $a\,b\,c\,d$ (fig. 1) will be subject to a shearing-strain of amount $\frac{p}{n}$; *i. e.* the angle $b\,a\,d$ will be diminished by an amount $\frac{p}{n}$, where p is the intensity of the shearing-strain on the element, and n is the coefficient of rigidity of the metal. The effect of this shearing-strain on the element may be described as a rotation of $a\,d$ through an angle s_1, and of $a\,b$ through an angle s_2, where $s_1 + s_2 = \frac{p}{n}$; s_1 and s_2 may be equal, when the displacement of c will be wholly parallel to $o\,x$; or they may be unequal, when c will have a displacement parallel to $o\,x$ and a resolved displacement parallel to $o\,y$, which will be +ve or −ve

according as s_1 is less than or greater than s_2, and which will be a maximum if either s_1 or $s_2 = 0$.

The value of $s_1 + s_2$, and the ratio of s_1 to s_2 for any element, appears to be determined by the following considerations. If a side $a\,b$ of the element be produced in both directions so as to cut the sides of the bar, there is seen to be a tendency of that part of the bar below this line to slide over that part which is above. Assuming the resulting shearing-strain to be uniform over the length l_1 of this oblique section, the shearing-strain at any element in this direction, *i. e.* s_1, will clearly be inversely proportional to l_1; s_2 will also be inversely proportional to l_2, where l_2 is the length of an oblique section parallel to $a'd$, and $s_1 + s_2$ will be proportional to $\frac{l_1 + l_2}{l_1 l_2}$—say to $\frac{1}{\rho}$.

Suppose, now, we take a series of elements, $a_1\,b_1\,c_1\,d_1$—$a_2\,b_2\,c_2\,d_2$ touching one another and lying across the bar. If we imagine the points a_1, a_2, a_3, &c. fixed on a line parallel to $o\,y$, it is clear that the displacements of c_1, c_2, c_3, &c. will be proportional to the sum of the shearing-strains in each element. If $s_1 + s_2$ is the same for each element, and $s_1 = s_2$, the points c_1, c_2, c_3, &c. will remain on a line parallel to $o\,y$. If, however, $s_1 + s_2$ becomes greater as we approach the middle, and less again towards the further side, the points c_1, c_2, c_3, &c. will be found on a line concave upwards; if $s_1 + s_2$ diminishes towards the centre, these points will lie on a line concave downwards. If the strains should exceed the limit of elasticity, the line through c_1, c_2, c_3, &c. will be permanently curved.

If the points a_1, a_2, a_3, ..., instead of being in one straight line have unequal downward displacements, the form of the curves obtained as above may be modified.

If, now, we consider a series of consecutive elements, $a_1\,b_1\,c_1\,d_1$—$a_1\,b_2\,c_2\,d_2$, lying in a line parallel to $o\,x$, if $s_1 = s_2$ for each element the points c_1, c_2, c_3, &c. will continue to lie on a line parallel to $o\,x$, since the displacement of each of these points, due to the distortion of the corresponding element, is parallel to $o\,x$. But if as we descend from the first element we find s_1 becoming greater than s_2, the points c_2, c_3, &c. will have a displacement to the left which increases as we descend,

so the line passing through c_1, c_2, c_3, &c. will be bent; and if the shearing-strains become equal again on the lowest element, this line will be concave to the right.

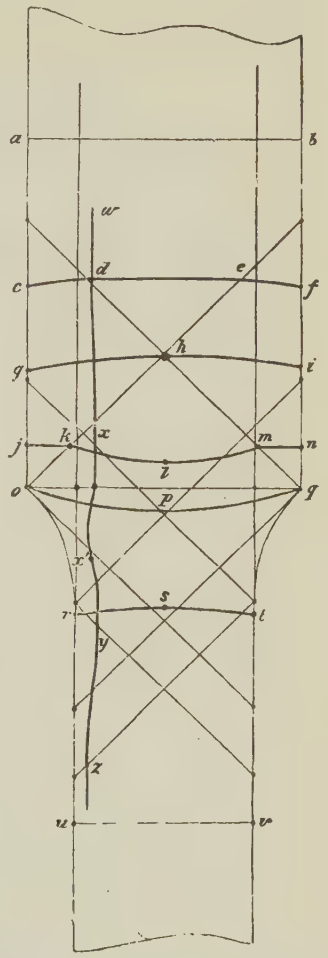
It thus appears that any horizontal line will be curved concave downwards if the sum of $s_1 + s_2$ is greater at the end of the line than in the centre, and *vice versâ*, but that the line will remain straight if $s_1 + s_2$ is constant along its length, and $s_1 = s_2$.

Also that a vertical line will remain so if s_1 is everywhere equal to s_2; but if s_1 is $> s_2$, the line will be curved and concave towards the right, and *vice versâ*.

We can now proceed to determine the curvature of the lines in the case before us.

Commencing at the upper extremity of the bar, fig. 2, a line such as $a\,b$ will remain straight, because ρ is everywhere constant, and $l_1 = l_2$. The line $c\,d\,e\,f$ will be curved concave downwards, since from c to d and from e to f, ρ is less than it is from d to e (where it is constant and $l_1 = l_2$); $d\,e$ will therefore remain straight, and $c\,d$ and $e\,f$ curved as shown. The line $g\,h\,i$ will be concave downwards and curved along its entire length, since ρ gets less as we move away from h on both sides.

Fig. 2.



The line $j\,k\,l\,m\,n$ will remain straight from j to k, since ρ is there constant and $l_1 = l_2$; from k to l, ρ becomes

less and greater again from l to m, $k\,l\,m$ is therefore concave upwards ; this line is straight from m to n, since ρ is constant and $l_1 = l_2$.

The line $o\ p\ q$ is curved concave upwards throughout the entire length, since ρ diminishes from o to p and increases from p to q.

The line $r\,s\,t$ will be concave downwards, since ρ diminishes from r to s and increases from s to t.

The line $u\ v$ will remain straight, since ρ is constant and $l_1 = l_2$

To turn now to the curvature of the vertical lines. If we consider one of these as $w\,x\,y\,z$, from w to d $l_1 = l_2$, at x l_2 is $< l_1$, while at x' $l_1 = l_2$ again, so that from d to x' this line will be curved in convex towards $o\,x$. Again, at y l_2 is $< l_1$, and at z $l_1 = l_2$, therefore $x'\,y\,z$ is also curved in convex towards $o\,x$.

Fig. 3.

It would appear then that the horizontal lines will have three distinct changes of curvature, while the vertical lines will be pinched in above and below the shoulder.

To test the accuracy of these conclusions, I prepared a copper bar as shown in the following way. The sides and faces were very carefully planed and polished, and on one face I inscribed a series of vertical and horizontal lines in a lathe. I then strained the bar so as to produce a considerable permanent set. An impression was then taken from the bar, which is reproduced in fig. 3*.

* The diagonal lines in fig. 3 were ruled on the impression to serve as lines of reference.

By placing the eye very nearly in the plane of the paper and looking along the lines, the curvatures can be seen to follow very closely those sketched on fig. 2. I would specially draw attention to the partial curvature of such lines as *c d e f* and *j k l m n*.

LV. *Galvanometers.* *By* Prof. W. E. Ayrton, *F.R.S.*, T. Mather, *and* W. E. Sumpner, *D.Sc.**

In order that a number of students may be able to work at the same time in the Physical Laboratories of the City and Guilds of London Central Institution we have endeavoured, as far as possible, to arrange the apparatus so that each of the many experiments should be complete in itself. This has led to the necessity of our constructing, and having constructed for us, a large number of galvanometers of various types, and from the specimens that are on the table it will be seen that several of the galvanometers contain points of novelty. A long series of comparative tests has been carried out with all the more important types that are in our laboratories, and we have therefore thought that a record of the results obtained by the students, and by ourselves, will be of value in affording data to guide others in the selection of instruments most suitable for the objects in view, as well as in suggesting future improvements in the manufacture of galvanometers.

I. *Astatic or Non-Astatic.*

One of the first questions that arises is whether it is desirable to employ the astatic type in the construction of sensitive galvanometers. Prof. A. Gray, in his treatise on 'The Theory and Practice of Absolute Measurements in Electricity and Magnetism,' maintains that it is not, for he says (page 311):—"Sensibility is sometimes obtained by the use of astatic galvanometers, but these are rarely necessary and are more troublesome to use than ordinary non-astatic instruments."

This opinion carries special weight as it may be very probably taken as expressing Sir William Thomson's view on this subject, seeing that the usual method adopted by Sir

* Read January 17, 1890.

William for measuring small currents is to employ a galvanometer with a single set of magnetic needles, and to weaken the magnetic controlling field as far as requisite by means of adjustable magnets. Theoretically there is no limit to the sensibility obtainable in this way; but with any strength of the resultant controlling field due to the earth and the adjustable magnets an astatic combination of magnetic needles must give a more sensitive arrangement than a non-astatic one. For let F be the resultant strength of this controlling field, let M be the magnetic moment of a needle within the coil whose magnetic constant is G, and let C_1 be the current flowing, then, if α be the deflexion, we have for small deflexions

$$\alpha = \frac{C_1 G}{F}.$$

Next, let a needle of moment m be attached to the suspended system outside the coil, so as to form an approximately astatic system with the needle inside, then, even if we neglect the action of the coil on the needle outside, an action which of course assists the action of the coil on a needle inside, we have, if C_2 be the current now flowing,

$$\alpha = \frac{C_2 G M.}{F(M-m)}.$$

Hence, if the deflexions be the same in the two cases,

$$C_2 = C_1 \frac{M-m}{M}.$$

Now M may be made large, and $M-m$ small, hence C_2 may be a small fraction of C_1. Consequently for any adjustment of the controlling magnet, the current required to produce a given deflexion can be much diminished by using an astatic combination instead of a non-astatic one.

But, perhaps, the most important advantage of an astatic over a non-astatic instrument is its much greater freedom from outside magnetic disturbance. In order that C_1, the current with the non-astatic galvanometer, may be small for a given value of the deflexion α, the resultant controlling field F must be small; whereas with the astatic combination there is no necessity to make F small in order that C_2 may be small,

since $\frac{M}{M-m}$ may be made large. Now the disturbance that an extraneous magnetic body can bring about depends on the strength of the magnetic field it sets up compared with that of the field F produced by the earth and the adjusting magnet combined. Hence the moving about of a magnetic body in the neighbourhood of the galvanometer will produce more disturbance on a non-astatic galvanometer than on an astatic galvanometer of the same sensibility. The preceding are very important considerations; hence, so far from agreeing with Prof. Gray that it is more troublesome to work with astatic galvanometers than with non-astatic ones, the preceding considerations, which we find fully borne out by experience, have led us to regularly employ the astatic principle with sensitive galvanometers.

And generally we may conclude that when it is desired to utilize a magnetic control, for example, with electrometers, &c., it is better, when a small controlling moment is required, to obtain this by the use of a weak magnet on the suspended system than by attempting to simply weaken the controlling field. Indeed, we may mention that even in the case of electrometers, where there is no question of the action of currents on magnets, and where (when a magnetic control has been employed in place of a bifilar or torsional control) it has been usual to attach a single little magnet to the aluminium electrometer-needle, to be acted on by a stationary outside magnet, we have found it more convenient to attach an *astatic* combination of magnets to the moving needle, and to direct the system by a fairly strong permanent magnet, which acts of course differentially on the suspended astatic system of magnets, since with such an arrangement great freedom from outside magnetic disturbance is secured, combined with the power of employing a wide range of sensibility.

It is worth noticing that in galvanometers, where the distance of the controlling magnet from the needles is varied by simply raising the magnet M, as in the case of the galvanometer seen in fig. 1, it is more convenient to place the galvanometer so that the plane of the coils is in the magnetic meridian of the laboratory when the controlling magnet is removed; or, in other words, to place the galvanometer so that

the spot of light comes to the zero of the scale for no current passing through the instrument and with no controlling magnet. For in that case the sensibility of the instrument can be subsequently varied by simply raising or lowering the

Fig. 1.

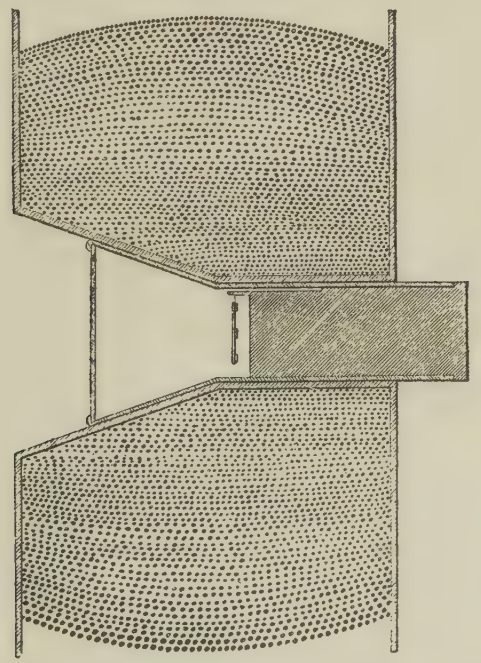
controlling magnet *parallel to itself*; whereas, if the galvanometer be fixed in any other position, it is necessary, when raising or lowering the controlling magnet, to give it a care-

ful screwing motion to prevent the spot of light going off the scale.

II. *Position of Mirror.*

According to the ordinary method of constructing a Thomson's reflecting-galvanometer, a tubular space at the centre of the coil is left unwound, and the mirror, with the magnets at the back of it, hung at the centre of this unwound space (see fig. 2, which illustrates a well-known form of this type of galvanometer). To allow the ray of light to pass out

Fig. 2.



of the coil after reflexion at the mirror, when the mirror is deflected, the end of the tubular space has to be made trumpet-shaped, which leads to still more space in the neighbourhood of the needle being left unwound. And when the galvanometer is an astatic one, a somewhat similar tubular space is left unwound on each side of the lower coil to allow room for the diamond-shaped aluminium vane to turn.

In this way the most valuable part of each coil, viz. that

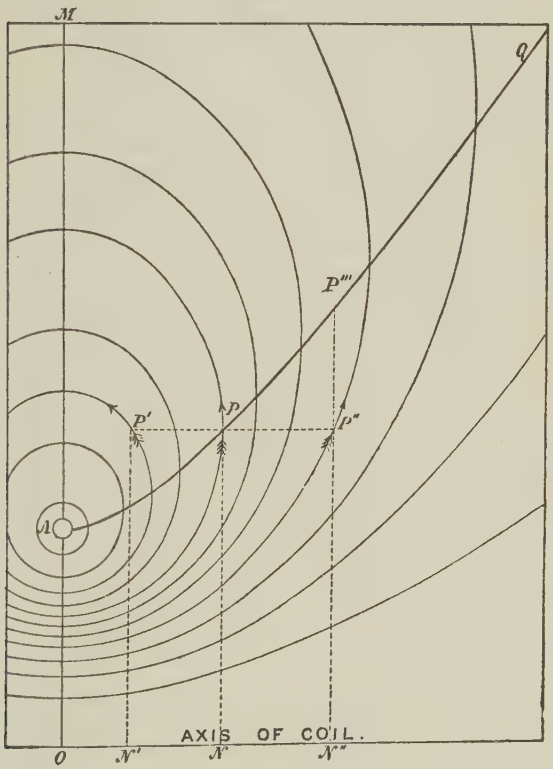
close to the magnetic needles, is left unwound. This disadvantageous mode of construction so impressed itself on Mr. Mudford, while a student at the City Guilds' Institute, that he suggested that the needles alone should be placed within the coil, and that the mirror (which is, of course, employed to reflect a ray of light, and not, as its position *inside* the coil would seem to infer, to be acted on electromagnetically) should be placed outside the coil, or between the upper and lower sets of coils when two pair of coils are employed. A specimen of a galvanometer constructed in this way and containing certain other improvements, which will be referred to later on, was submitted some years ago by Mr. Mudford to Sir William Thomson, who expressed approval of the devices employed, and, since that time, many galvanometers differing in other details, but all embodying the principle of not wasting valuable space in the coils, have been constructed for the Central Institution.

It is interesting to notice that the common method of placing the mirror inside the coil is an irrational survival of an old custom kept up, like the two buttons at the back of a coat, when its use is almost forgotten. Gauss and Weber put their mirror inside the coil because the mirror was made of polished steel and was also the magnetic needle. Sir William Thomson put the mirror inside the coil in his "speaking galvanometers," for receiving messages on submarine cables, because he desired to render the space near the mirror as air-tight as possible in order to obtain great damping. And the ordinary maker of astatic galvanometers puts the mirror inside the coil, because he has seen it there in other galvanometers, and it has never occurred to him to put it anywhere else.

If the ray of light has not to go into the coil no hole need be left and the coil may be wound practically to the centre. It is necessary, however, that the space left unwound inside the coil should be somewhat larger than is required simply to allow the needle to turn through a small angle, because if the windings were carried to the axis of the coil in the immediate neighbourhood of the needle the effect on the latter of the current passing through the innermost coils would be opposed to that of the rest of the coil. To find the shape and volume of the least space that should be left unwound we proceeded as follows:—

Let OA (fig. 3) be the radius of a convolution of wire whose plane is perpendicular to the paper, the section of the convolution being represented by the small circle at A, then, from

Fig. 3.



Maxwell's 'Elect. and Mag.' vol. ii. plate xviii., the lines of force due to a current passing round this convolution are as indicated. Therefore, if a magnet whose half length is NP be placed at a distance ON from the plane of the convolution such that NP is a tangent to the line of force at P, no torque will be exerted on the magnet by a current passing round the convolution, since the direction of the force at P is along the magnet NP. But if the magnet be placed nearer to the convolution, as at N′P′, or farther from it, as at N″P″, there will be a torque exerted on it due to this convolution of radius OA ; the two torques, however, will be in opposite directions.

Now as the radii of the greater number of convolutions contained in the coil are greater than the half length of the magnet, it is clear that the torque exerted by the convolution OA when the magnet is to the right of NP is in the same direction as that exerted by the coil as a whole, and that the torque exerted on the magnet by this convolution when the magnet is between OA and NP is opposed to the torque exerted by the rest of the coil. In other words, if NP be the position of the half-magnet in the coil, a convolution of smaller radius than OA must not be wound in the plane that passes through OAM at right angles to the paper. And if a line APQ be drawn so as to cut all the lines of force at points where the tangents are perpendicular to the axis of the coil, OA gives the smallest radius that a coil must have in the plane passing through OAM at right angles to the paper, whether the magnet be at N and its half length be NP, or the magnet be at N″ and its half length be N″P‴.

For our purpose, however, we have not to consider the problem of magnetic needles of different lengths at different distances from a fixed plane, but the converse problem of a magnetic needle of a fixed length in a fixed position, which is at different distances from different planes, the critical radius of convolution in each of which we wish to determine.

If NP were the fixed half length of the magnet, then, as already seen, OA would be the critical radius of a convolution at a distance NO from N, and the critical radius of a convolution for another distance can be obtained by imagining the figure 3 reduced in the proportion of NP to N″P‴, in which case a critical radius $\frac{NP}{N''P'''} \times OA$ will be obtained at a distance $\frac{NP}{N''P'''} ON''$ from N. Or, lastly, if we take OA as the half length and position of our magnetic needle, the critical radius of convolution at every distance from O will be obtained by taking every such ordinate as N″P‴ and multiplying OA by the ratio of OA to N″P‴, and the distance from O for which this is the critical radius is equal to $\frac{OA}{N''P'''} \times ON''$

A curve giving the locus of such critical radii of convolution is shown in fig. 4 for a needle of length 2, shown therefore in

the figure about twelve times its full size, and we learn from it that at a distance from the centre of the coil equal, say, to 0·4 of the half length of the needle, the smallest convolution should have a radius about 0·75 of the half length of the needle, and that the wire must not be wound close to the axis until the distance from the centre along the axis is about 0·72 of the half length of the needle.

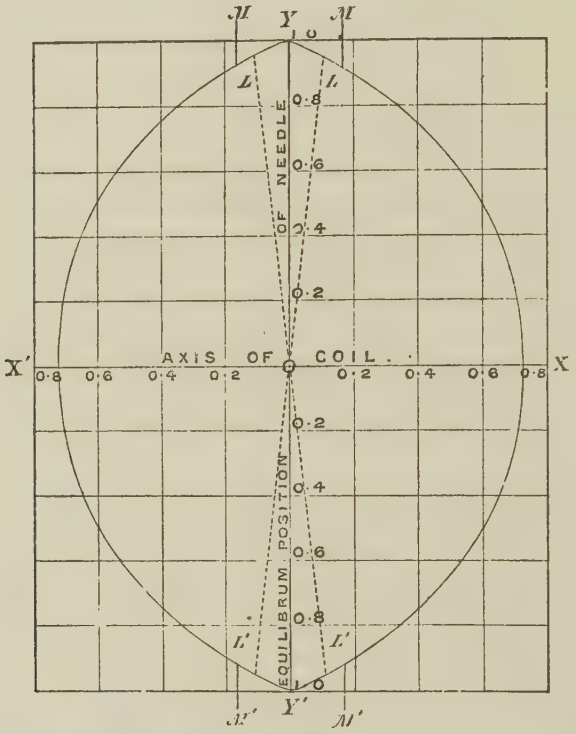
With an ordinary reflecting-galvanometer the needle requires to have a free angular space for turning of about 15°, represented by the space between the two lines LOL′. In this region the radii of the smallest convolutions must be a little greater than theory allows, otherwise the needle would touch the coils, and generally a sufficiently near approximation can be made to the cavity which theoretically ought to be left unwound by making it an oblate spheroid with a polar axis about 0·72 of its equatorial diameter, the latter being of course slightly larger than the length of the needle.

Since winding wire inside the surface whose section is given in fig. 4 would oppose the effect of the wire wound outside this surface if the current flowed in the same sense throughout the whole coil, it is possible, by causing the current to circulate in opposite directions in the two portions, to wind all the space and to cause all the convolutions to help one another. This we have not yet tried, but in view of the fact that the space which has been left unwound in the galvanometers constructed according to Mr. Mudford's suggestion is very near the needle, and therefore very valuable, it seems important to try and utilize it in the way just suggested. In that case the surface we have theoretically determined would be the surface separating the coil into two parts, wound respectively in opposite directions.

As the coils of a reflecting-galvanometer have to be supported by a central framework, there is necessarily a certain distance left between the coils when they are in position. This space we find when drawn to scale is that contained between the two vertical lines MM′, and which therefore allows more than sufficient room for the needle to turn. Hence if the plan of opposite winding that we have suggested be tried, the face of the coils may be made quite plane and no portion

whatever of the coils left unwound, even to allow the needle space to move.

Fig. 4.



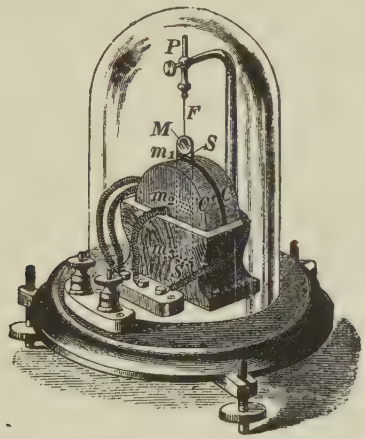
Shape of cavity to be left unwound inside the coil when the length of the magnetic needle is 2.

III. *Damping the Oscillations.*

Since no hole is left in the lower coil for the aluminium damper, as is the case with Messrs. Elliott's reflecting-galvanometers, a different mode of damping is adopted. The plan we find best is to attach the mirror M (fig. 5) to the middle of a long vertical strip of mica, S, across the upper and lower ends and the middle of which the magnets m_1, m_3, and m_2 are fixed. Such an arrangement is also seen in position in the galvanometer shown in figure 6 later on. With this arrangement, which requires very little space, we find that when the controlling magnet is adjusted so

that the periodic time of vibration is 7 seconds or more, the spot of light does not visibly pass through the zero

Fig. 5.



or the instrument is dead-beat, and even when the period is much shorter than this, the spot of light does not pass through the zero more than twice*. With galvanometers in which less damping is desired, a portion of the strip of mica is cut away, so as to make it narrower, and in extreme cases, when for example no damping is required, as in the case of a ballistic galvanometer, the mica strip is replaced by a vertical piece of wire.

With a ballistic galvanometer, especially when intended for teaching-purposes, it is desirable to have some easy means of adjusting the amount of damping. In the case of the galvanometer used by our students for experimenting on the variation produced in the swing of the needle of a ballistic galvanometer by variations in the amount of damping, this variable damping is effected by enclosing the mirror in a cell whose glass ends can be simultaneously made to approach or recede. This motion of the ends of the cell is effected by

* Quickness in the needle coming to rest, which is produced by using a powerful controlling field, must not, of course, be confounded with considerable damping, since, other things being the same, the decrement is the greater the weaker the controlling field and the slower the motion of the needle.

turning a milled head outside the instrument which works a right-and-left-handed screw inside.

In order to correct the throw of a ballistic galvanometer for the damping of the oscillations of the needle, it is well-known that it is necessary to multiply the sine of half the angle of throw by the expression

$$V = \frac{1}{\sqrt{1+\frac{\lambda^2}{\pi^2}}} \Sigma \frac{\lambda}{\pi} \tan^{-1} \frac{\pi}{\lambda},$$

where λ is the logarithmic decrement of the oscillations. The value of this quantity when λ is so small that its square may be neglected is

$$V = 1 + \tfrac{1}{2}\lambda.$$

It seemed, however, desirable to work out the value of V more fully and also to express it in terms of the numerical ratio of each amplitude to its successor, since in this way the need of finding the Napierian logarithm is avoided. This we did at first by direct calculation, but while forming a table of values connecting V and the decrement we observed that the matter could be treated much more simply as follows :—

By taking logarithms of the complete expression for V we have

$$\log V = -\tfrac{1}{2} \log (1+x^2) + x \tan^{-1} \frac{1}{x},$$

where

$$x = \frac{\lambda}{\pi}.$$

Differentiate both sides of this equation once, twice, and thrice with respect to x and substitute for x the value 0. We then obtain successively,

$$\frac{dV}{dx} = \frac{\pi}{2}; \quad \frac{d^2V}{dx^2} = \frac{\pi^2}{4} - 3; \quad \frac{d^3V}{dx^3} = \frac{1}{8}\pi^3 - \frac{9}{2}\pi.$$

By Maclaurin's theorem we then have

$$V = 1 + \frac{\pi}{2}x + \left(\frac{\pi^2}{4} - 3\right)\frac{x^2}{2} + \left(\frac{1}{8}\pi^3 - \frac{9}{2}\pi\right)\frac{x^3}{6} + \&c.;$$

or substituting for x, and working out numerically, we obtain

$$V = 1 + 0{\cdot}5\lambda - 0{\cdot}027\lambda^2 - 0{\cdot}054\lambda^3,$$

which shows that until λ approaches unity the approximate value for V generally used is sufficiently accurate. Now if the ratio of the amplitude of any swing to that of its successor is $(1+y)$, we have

$$\lambda = \log(1+y) = y - \frac{y^2}{2} + \frac{y^3}{3} - \&c.,$$
$$\lambda^2 = y^2 - y^3,$$
$$\lambda^3 = y^3.$$

Neglecting powers higher than the third, we obtain by substitution

$$V = 1 + 0{\cdot}5y - 0{\cdot}277\,y^2 + 0{\cdot}130y^3,$$

from which we can at once calculate the value of the complete correcting factor when we have observed the decrement.

It is usual to determine the sensibility of a ballistic galvanometer either by the employment of an earth-inductor, or by discharging through the galvanometer a condenser charged to a known P.D. The first method, however, necessitates an *exact* knowledge of the horizontal or vertical component of the magnetic intensity at the spot, while the second requires an *exact* knowledge of the value of the capacity of the condenser and of the P.D. employed. But now that it is possible to obtain an ammeter calibrated with a high degree of accuracy, the simplest method of determining the sensibility of a ballistic galvanometer is to first calibrate it absolutely as a galvanometer for measuring currents by direct comparison with the ammeter. Its absolute calibration as a ballistic galvanometer can then be determined from the fact that

The swing per micro-coulomb in scale-divisions

$$= \text{the deflexion per micro-ampere} \times \frac{2\pi}{T},$$

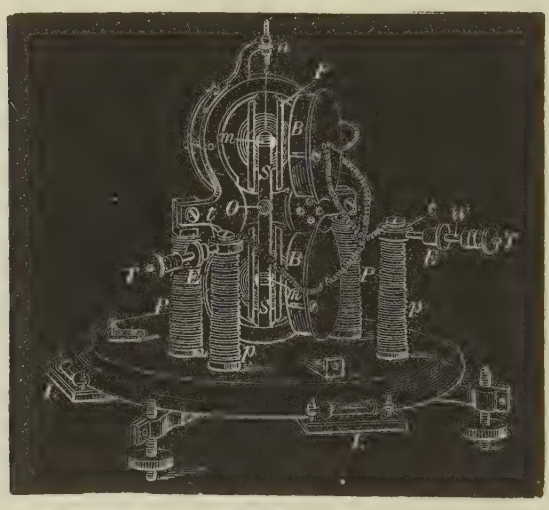
where T is the periodic time of oscillation of the needle in seconds.

IV. *Supporting the Coils.*

In olden days Messrs. Elliott supported the coils in their galvanometers by simply employing three long screws to squeeze each coil between a brass plate and the framework of the instrument to which the plate was screwed. The disadvantage, however, of such an arrangement was made painfully

clear in the use of a galvanometer constructed by this firm for one of the authors in 1868. For on transporting this galvanometer across India, soon after it was made, the brass screws expanded more than the coils, and the coils consequently slipped down, shearing away the needles, the mirror, and the aluminium vane. It was therefore suggested to Messrs. Elliott that the coils should be supported in boxes B, preferably hinged, as shown in fig. 6, so that when the coil-boxes are opened, as

Fig. 6.



in this figure, the suspended system S could be got at and even removed without detaching the connecting wires from the coils. The galvanometer shown in fig. 6 is, when in use, covered up by the brass case supporting the controlling magnet, illustrated in fig. 1.

Hinged boxes containing the coils of a reflecting-galvanometer appear to have been employed even earlier than this by Mr. Pepper, at the Polytechnic, but that form of construction was, apparently, in 1868 unknown to, or at any rate unused by, instrument makers.

In the case of galvanometers intended to be highly insulated, these hinged boxes may with great advantage be made of ebonite, and to prevent the boxes becoming electrified and

acting on the suspended needle electrostatically, one terminal of the galvanometer should be joined to the outside brass case.

In the galvanometer shown in fig. 6, the boxes are kept closed by a screw which is screwed into the framework of the instrument, but in the galvanometer shown in fig. 7 this screw is replaced by a spring button, which is a more convenient arrangement.

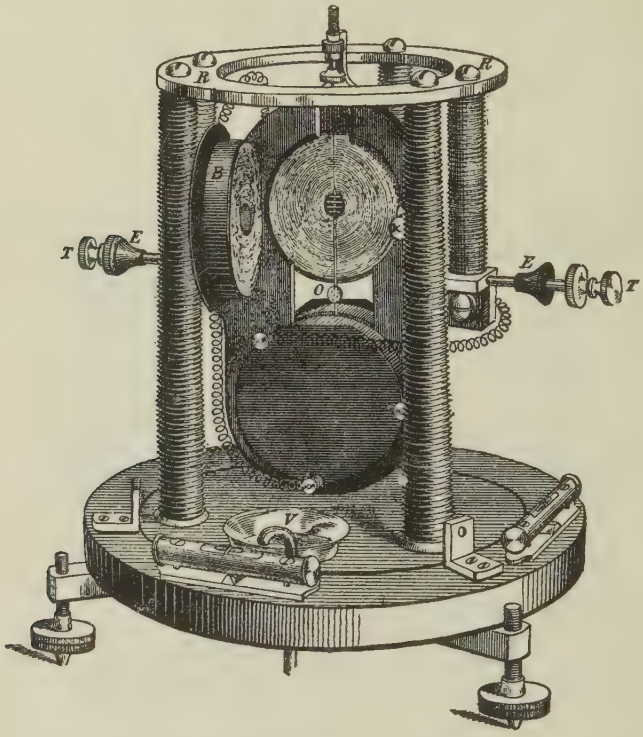
V. *Insulation of Coils and Terminals.*

In the ordinary method of constructing reflecting-galvanometers the wires are brought under the base to terminals fastened on to it. But such an arrangement is most unsatisfactory, since, no matter how thoroughly the upper part of the ebonite base may be cleaned and dried, leakage is almost sure to take place between the two wires, between the terminals, and between both the wires and the terminals to the ground along the inaccessible under surface of the ebonite base. In fact, when one of the authors was engaged, on behalf of the Indian Government, in 1872 testing telegraph insulators at Messrs. Siemens's works at Charlton, it was the common practice every morning to light a gas-burner near the galvanometer to dry it, and to diminish the leakage to a workable amount. And it appears to the authors that the form of expensive astatic reflecting-galvanometer, that is still made by some firms, with the wires underneath the base, is but a survival of the antiquated type of leaky electrostatical apparatus, to use which required that the whole air of the room should be first dried. Fig. 6 shows the obvious way of overcoming this difficulty, and which consists in supporting the framework and the coils of the apparatus as well as the terminals on paraffined corrugated ebonite pillars *p*, which are maintained clean and dry by their being kept entirely inside the brass cover of the instrument Q (fig. 1). The terminals T at the ends of the wires W project through holes H in the brass case, these holes being kept closed by tightly-fitting ebonite collars C, fig. 1, E, fig. 6, when the galvanometer is not in use, or when it is being employed for some test where a small amount of leakage is not important.

Still higher insulation is obtained with the galvanometer

shown in fig. 7, since in this instrument the coils are supported from two corrugated ebonite rods which hang from

Fig. 7.



Very high resistance galvanometer with brass case removed.

a brass ring R, carried on the top of three corrugated ebonite pillars fixed to the slate base-plate. This instrument, the four coils of which have a joint resistance of nearly 400,000 ohms, was constructed by Messrs. Nalder Bros.; but the device, by means of which the shortest path along which surface-leakage can take place from the coils, or from the terminals, to the base of the instrument is between 30 and 40 inches of ebonite artificially dried by sulphuric acid, is due to Messrs. Eidsforth and Mudford. To prevent any possible overflowing of the sulphuric-acid vessel, a syphon passes through the base on the Tantalus cup principle, and which empties the liquid when it rises to the level of the bend of the syphon.

VI. *Proportionality of Deflexion and Current.*

Some authorities are of opinion that it is of comparatively little consequence what is the law connecting deflexion and current, since for very accurate work any galvanometer must be carefully calibrated experimentally, and the exact law connecting deflexion and current thus ascertained. But, on the contrary, although the necessity of calibration for accurate work is perfectly true, experience has shown us that for ordinary practical work much time is saved if the readings on the scale are approximately, even if not quite accurately, proportional to the currents producing them. This latter result can, of course, be obtained either by fitting the lengths of the divisions on the scale to the peculiarities of the galvanometer, or by starting with a uniformly divided scale and constructing the galvanometer in such a way that equal additions to the current produce equal angular deflexions.

In certain cases, when, for example, the galvanometer is mainly employed for measuring currents all having about one definite value, as for example in the case of a voltmeter used on an electric-lighting circuit, it is obviously desirable to have the scale widely extended at the part where it is most used, and to obtain this result it is better to have the divisions crowded together elsewhere. But, since the power of accurately subdividing the spaces on a scale by eye is much increased if all the spaces be of equal length, it is preferable, in the case of galvanometers for general use, and when all parts of the scale are equally valuable, to judiciously construct the galvanometer in such a way that the whole scale may be divided into equal distances which are directly proportional to equal increments of current, than to give the coils and needle a shape arrived at in a haphazard fashion, and then attempt to experimentally subdivide the scale to suit the vagaries of the galvanometer.

In a former paper communicated by Prof. Perry and one of the authors to this Society, it was pointed out that while a galvanometer may be a "proportional" instrument if the zero for no current be that corresponding with the needles being in a symmetrical position to the coils, the same instrument may be very far from proportional if the zero for no current

be taken at one end of the scale and the spot of light deflected right across the scale for increasing currents, since in this latter case a variation in the law occurs as the plane of the needles passes through the symmetrical position. This want of proportionality is exhibited by the following figures :—

Ordinary Reflecting-Galvanometer.

Deflexion from one end of the Scale.	Relative Strength of Current.	Error.	Percentage Error.
0	0	0	
60	62·0	+2·0	3·4
120	121·0	+1·0	0·8
180	181·5	+1·5	0·9
240	242·5	+2·5	1·0
300	303·0	+3·0	1·0
360	363·5	+3·5	1·0
420	423·5	+3·5	0·8
480	483·0	+3·0	0·7
540	542·0	+2·0	0·4
600	600·0	0	

This defect is even more marked in some parts of the scale with the ordinary d'Arsonval galvanometer when the zero for no current is at one end of the scale. This instrument, as usually made by Messrs. Carpentier, has a tubular iron core supported from a standard which carries the top suspension of the coil, and the stationary permanent magnets are not provided with pole-pieces. In consequence of the core and the suspension not being supported independently, there is great difficulty in centering the coil relatively to both the core and the stationary magnets ; further, neither the core nor the coil can be readily removed. And partly because of this difficulty of centering the coil, partly because its centre of gravity is generally not in the line of suspensions, and partly because the stationary magnets have no properly shaped pole-pieces, there is (as may be seen from the following sample of the results of many experiments we have made on d'Arsonval galvanometers) an error in different parts of the scale far less regular in its value than in the case of the ordinary reflecting-galvanometer with stationary coil.

Ordinary d'Arsonval Galvanometer.

Deflexion from one end of Scale.	Relative Strength of Current.	Error.	Percentage Error.
0	0	0	
60	59	−1	−1·7
120	118·5	−1·5	−1·3
180	180	0	0
240	240·6	+0·6	+0·2
300	300	0	0
360	359·8	−0·2	−0·06
420	416·5	−3·5	−0·8
480	473·5	−6·5	−1·4
540	537	−3	−0·6
600	600	0	

We find that it is possible, however, to obtain with a d'Arsonval galvanometer a far more exact proportionality between the deflexion and the current than is given in the preceding table, by:—

Supporting the coil independently of the iron core, so that fairly accurate centering becomes possible. This result has been to a certain extent accomplished in the small type of d'Arsonval galvanometer constructed by Messrs. Nalder.

Allowing the coil to hang freely from the top suspension so that the centre of gravity of the coil is always in the axis of rotation. Electric continuity at the bottom of the coil is then maintained by a spiral of fine wire offering little torsional rigidity.

Fitting iron pole-pieces to the stationary magnets, and shaping these pole-pieces so that the rate of cutting lines of force by the moving coil is practically constant.

The following gives the results of tests made on a d'Arsonval galvanometer constructed in this way, each divisional length on the scale being 1·05 millim., so that the total deflexion of 600 divisions represented a motion of the spot of light over about 63 centimetres, or about 25 inches. The deflexion could be read easily to 0·2 of a division, and it will be observed that the greatest want of proportionality is only 0·15 per cent., which occurs with a deflexion of 480 scale-divisions.

Improved d'Arsonval Galvanometer.

Deflexion from one end of Scale.	Relative Strength of Current.	Error.	Maximum Percentage Error.
0	0	0	
60	60	0	
120	119·9	−0·1	
180	179·8	−0·2	
240	239·9	−0·1	
300	299·8	−0·2	
360	360·2	+0·2	
420	420·5	+0·5	
480	480·7	+0·7	0·15
540	540·5	+0·5	
600	600	0	

To improve the magnetic circuit in d'Arsonval galvanometers some English makers replace the original tubular core by a solid iron core; and Messrs. Jolin of Bristol form the core of a series of short horizontal magnets laid vertically one above the other. A comparison that we have made of the sensibilities of the instruments constructed in these different ways has not shown that the English instruments are superior in this particular respect to the French. This may be, however, due to the fact that the horseshoe permanent magnets employed by the English manufacturers are inferior to those used by Messrs. Carpentier.

The ordinary methods adopted for attaching the suspension wires of the d'Arsonval coil to the supports seem to have been designed with total disregard of the fact that these wires convey the current into and out of the coil. This is particularly serious when a d'Arsonval galvanometer is used as a voltmeter, especially when it is employed to measure a small fraction of a volt, and when therefore the total resistance in the circuit must be small.

In fact we have found that before any reliance can be placed on the indications of a d'Arsonval galvanometer employed in this way, it is necessary to solder all the joints. And instead of trusting to contact through the supports to which the torsional suspension-wires are attached, we have found it desirable to solder wires coming from the terminals of the instrument directly to the hooks to which the suspension-wires are attached. Perhaps the best plan is to solder

wires at one of their ends to the terminals of the instrument, and at their other ends to an extension of the torsional suspension-wires.

In devising very low-resistance d'Arsonval galvanometers careful consideration must be given to the suspension-wires which lead the current into and out of the moving coil, since being usually made of German silver they may easily become slightly heated by the relatively large current employed with a very low resistance-galvanometer, and in consequence their elasticity temporarily diminished. This would of course have the effect of making the instrument more sensitive for large currents than for small; but as the sensibility for large currents would, until a limit was reached, increase with the time the current was kept flowing through the galvanometer, such an instrument could not be used for accurate measurement. One method of overcoming this difficulty is to make the suspension of very thin phosphor-bronze strip, such as is employed in the construction of delicate "Ayrton and Perry magnifying-springs." For a thin strip has, for its cross-section, a very small torsional rigidity and a very large radiating surface, and therefore is the very thing to employ when we want a conductor with small torsional rigidity, and which will be very slightly raised in temperature by the passage of a current through it.

D'Arsonval galvanometers, when used as voltmeters, are subject to the error that affects all electromagnetic voltmeters, arising from a variation of the resistance of the coil due to a variation in the temperature of the room; if, however, both the coil and the suspension-wires are made of platinum-silver, a very curious compensating effect is brought about, since the percentage increase of resistance with temperature of platinum-silver is almost exactly equal to the percentage diminution in its torsional rigidity. Hence when a rise of temperature diminishes the current flowing through the instrument for a given P.D. maintained at its terminals and so diminishes the deflecting couple, a proportional diminution in the torsional rigidity of the suspending wires, and therefore in the controlling couple, is produced at the same time. *It is thus possible to construct an electromagnetic voltmeter of one metal only, which has no temperature error.*

VII. *Sensibilities of Different Types of Galvanometers.*

Apart from differences in detail (as, for example, in the size shape, or number of the coils, &c.), galvanometers may be divided into four distinct classes, viz.:—

1. Galvanometers in which the moving system consists of one or more magnetic needles turning about an axis at right angles to the magnetic axes of the needles.
2. Galvanometers in which the needles have a motion of translation along their own magnetic axes, the magnetic needles being drawn into, or pushed out of, coils acting like solenoids.
3. Galvanometers in which the magnetic system is stationary, and the moving system consists of a coil turning about an axis on its own plane.
4. Instruments based on the heating of a conductor by the passage of a current through it.

Type 1 includes tangent-galvanometers, Thomson's reflecting-galvanometers—in fact, so many well-known instruments that further description is unnecessary. In addition to galvanometers with straight magnetic needles, it may be noticed that type 1 also includes instruments containing a bell-shaped horseshoe-magnet; since the bell-shaped magnet is only a device for producing the equivalent of a very powerful and permanent short flat magnet.

We may here mention that the instrument shown in fig. 5 is spoken of in this paper as a galvanometer with "one pair of coils," while those illustrated in figs. 6 and 7 are referred to as having each "two pairs of coils."

Type 2 has been employed by Lord Rayleigh, the Profs. Gray, Mr. Rosenthal, by ourselves, and others. Figs. 8 and 9 illustrate an instrument of this type, constructed by Messrs. Edelmann of Munich, and called a Rosenthal Micro-galvanometer, since this instrument has the peculiarity that the coils into which the ends of the magnet are sucked are extremely small, having an *outside* diameter of only 11 mm. and a thickness of 2 mm. Fig. 9 shows the suspended magnet of the Rosenthal galvanometer as constructed by the makers, the arrangement being non-astatic; but we have also made and used with this galvanometer an astatic combination, constructed as in fig. 10.

Fig. 8.

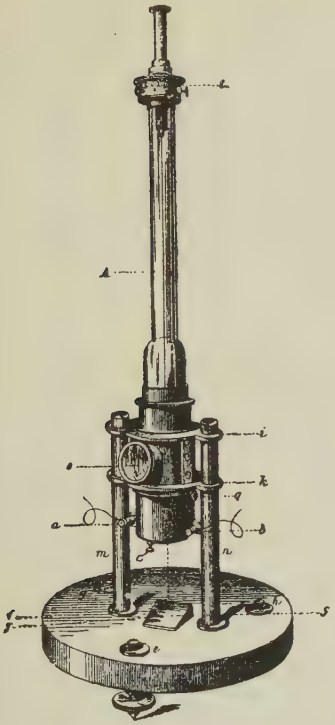


Fig. 9.

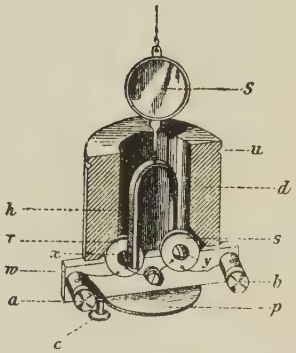
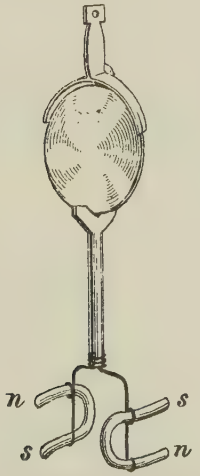


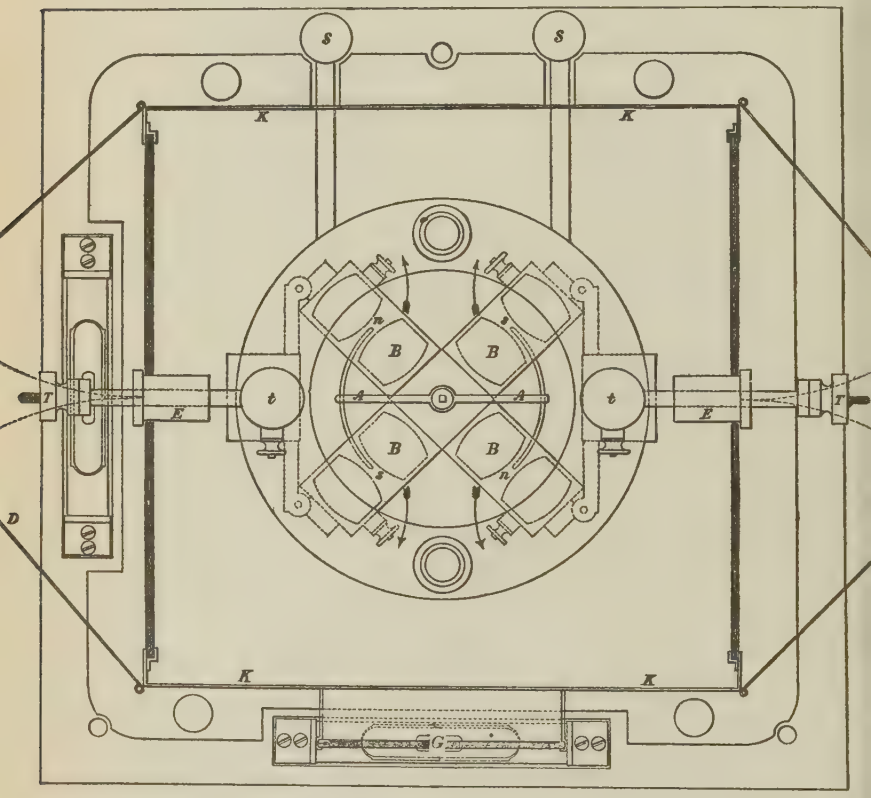
Fig. 10.



Mirror and astatic pair of needles for the Rosenthal Micro-galvanometer.

Figs. 11 and 12 show, in plan and elevation, a ballistic galvanometer constructed for the Central Institution by Messrs. White of Glasgow, from drawings made by us in consultation with Prof. T. Gray. The needles *ns*, *ns*, which are arranged so as to produce an astatic combination, are each at one end sucked into a coil and at the other end pushed out of a coil, the coils being contained in four small, rectangular, hinged boxes, B, which are kept closed by spring catches. The vertical aluminium wire, to which is screwed the aluminium bridge A A (fig. 11), supporting the needles, carries on one side

Fig. 11.

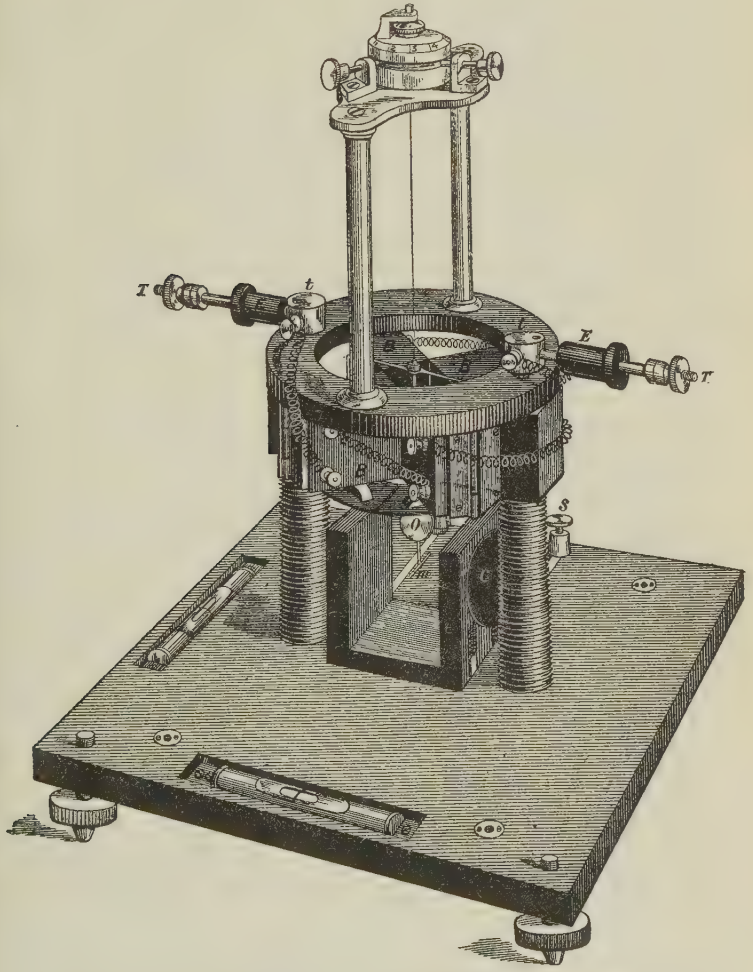


Plan of Ballistic Galvanometer.

of its lower end a concave mirror O (fig. 12) to produce a spot of light on a scale, and on its other side a truly

plane mirror, to be viewed with a telescope, if desired. At its lower end it carries a small magnetic needle, m,

Fig. 12.



Perspective elevation of Ballistic Galvanometer with brass case removed

to be acted on by a pair of coils, C, joined up in series and having their ends led to the terminal screws, s, s. These coils are quite independent of the main coils of the instrument, and are employed for damping the oscil-

lations of the suspended system. With ballistic galvanometers previously made for us we have not had any such damping-coil attached; but since to all our galvanometers which are used ballistically it has been found convenient to provide a damping-coil, operated with an auxiliary small cell and roughly-made reversing-key, for the purpose of expediting measurements, we think that such a damping-coil should be attached, by the makers, to every ballistic galvanometer.

The brass case, K, has hinged brass doors D, D, D, D at its sides, to protect the glass windows; and both doors and windows have holes in them to allow the terminals T, with their ebonite collars E, to be screwed into the brass terminal blocks, t, t. At the front of the instrument there is a circular plate-glass window G, to allow the light to pass through, and protected by a brass cap when not in use.

Type 3 includes the well-known d'Arsonval galvanometers, with the modification described to this Society some two years ago for obtaining "invariable sensibility," by replacing the ordinary torsional suspension with one having extremely small torsional rigidity and inserting a number of small magnets in the moving coil, which, being attracted by the large stationary magnet, produce the control.

Type 4 we do not propose touching on in this paper, as that type of instrument formed the subject of communications previously made to this Society by Prof. Perry and by one of the authors on "Hot-Wire Voltmeters" and "Twisted Strip Voltmeters."

It is important that the results of the numerous tests on many specimens of each of the first three types of galvanometer that have been carried out in the laboratories of the Central Institution, as well as the tests that have been made elsewhere, and the results of which we have collected together, should be reduced to the same standard; so that the relative merits of the different types of galvanometer, as well as of the different specimens of each type, should be able to be usefully compared. A satisfactory standard of comparison, however, cannot easily be arrived at; and we are not aware that the question of comparing galvanometers of totally different types has previously received much attention. The "figure of merit" of a galvanometer is sometimes defined as "the

amount of current which will produce one division or degree of deflexion"*. But although the figure of merit, so defined, tells us the sensibility of a particular galvanometer with a particular adjustment of the controlling magnet, and with a particular distance of the scale, it gives us no means of comparing this galvanometer with other forms, since this figure of merit can be varied by altering the position of the controlling magnet &c.—in fact, is not a constant of the particular galvanometer. Something much more definite than this is therefore required for our purpose.

A little consideration shows that the sensibility of a galvanometer in actual practice is made up of many factors, some of which depend on the skill of the experimenter himself. In view of the sensibility of any particular galvanometer being a complex result depending on many factors, it is desirable to record the results of experiments on galvanometers in such a form that the influence of each factor can be separately estimated. We have therefore endeavoured to distinguish, as far as possible, the three chief causes that affect the sensibility of reflecting-galvanometers, viz. the arrangement and winding of the coils, the construction of the magnetic system, and the optical method of magnifying the angular deflexion produced. With the same view the question of the susceptibility of various galvanometers to outside magnetic disturbances has been considered separately, since, first, the electrical qualities of the instrument are not altered by such disturbances, and, secondly, their influence is dependent, not only on the kind of instrument, but on the place of test. In the Table that is given later on recording the results of a large number of experiments, we have classified galvanometers in four divisions of decreasing susceptibility to outside magnetic disturbance, comprising :—

Non-astatic instruments working in a *weak* field having *strongly magnetized* needles.

Non-astatic instruments working in a *strong* field and having somewhat *weakly magnetized* needles.

Astatic galvanometers, working in a fairly strong field.

Galvanometers of the d'Arsonval type, working in an extremely strong field.

* 'A Handbook of Electrical Testing,' by H. R. Kempe.

At the end of the Table are given the constants of a collection of non-reflecting instruments. These instruments, being all of a rougher type, have been simply grouped together at the end of the list; consequently their position on the list has no reference to their sensitiveness or unsensitiveness to outside magnetic disturbance.

The optical sensitiveness of an instrument is dependent on many conditions that have no necessary connexion with the electromagnetic qualities of the instrument; for example, the goodness of the mirror, the brightness of the light, the distance of the scale, &c. are important factors. We have therefore given the number of scale-divisions per micro-ampere or per micro-coulomb, assuming the scale to be placed at a distance from the mirror equal to 2000 scale-divisions. As the scale-distance most generally used is a metre, and as the scale-divisions are frequently in half-millimetres, the conditions assumed are often fulfilled in practice. The sensibilities for a steady current, and for a discharge, are thus directly expressed in angular measure, irrespective of optical magnification.

An important and difficult question to decide is whether the factor of merit of a galvanometer should be stated for a constant periodic time of oscillation of the suspended system, or for a constant controlling-moment for unit angle-deflexion, or for a constant controlling-field irrespective of the magnetic moment of the suspended system. The first condition is the most simple to realize experimentally with different instruments, and the results so obtained are the most convenient for use in subsequent calculations; this condition is, further, the one most easily understood. We therefore give, in columns 3 and 4, the values of D and S, which respectively denote the deflexion per micro-ampere and the swing per micro-coulomb, each in scale-divisions, when the period is 10 seconds and the scale-distance is 2000 scale-divisions. The corresponding values at any other period T are of course $DT^2/100$ and $ST/10$. It would, however, be unfair to rest satisfied with the results given in these tables for the purpose of comparing different galvanometers. For a galvanometer having a large mirror, and intended to be used at a considerable distance from the scale, would appear to be an inferior instrument, because on

the one hand the large moment of inertia of the mirror would necessitate the controlling-moment being large to reduce the period to 10 seconds, while, on the other hand, the magnification arising from the much greater distance that separated, in practice, the scale from this particular galvanometer would have no weight in the values given in the columns for D and S. We have therefore given in column 2 the values of M, which are the millimetres deflexion per micro-ampere when the period is 10 seconds and the scale placed as in the ordinary use of the instrument. We have further, in the following way, reduced the numbers to their corresponding values under the condition of *constant controlling-moment*, the condition of constant controlling-field being rejected, because it is inapplicable to instruments of the d'Arsonval type.

The periodic time T equals

$$2\pi\sqrt{\frac{\text{Moment of inertia} \times \sin\theta}{\text{Controlling-moment for a deflexion } \theta}};$$

therefore, in order to obtain the comparative deflexions that would be obtained with the same controlling-moment per unit angle, we must find the deflexions that would be produced with periodic times proportional to $\sqrt{I}$, where I is the moment of inertia of the suspended system, the values of which for the various galvanometers are given approximately in C.G.S. units in column 6. Hence, to obtain the values in columns 7 and 8, which give numbers respectively proportional to the steady deflexion per micro-ampere and the first swing per micro-coulomb for the same controlling-moments for all galvanometers, we must multiply D by I and S by $\sqrt{I}$. Now the stability of the zero of any galvanometer depends on the controlling-moment per unit angle of deflexion ; hence columns 7 and 8 give numbers proportional to the angular deflexion per micro-ampere and the angular swing per micro-coulomb for the same amount of stability of the zero for all the instruments. The periodic times, however, will be very different for the different numbers either in column 7 or for the different numbers in column 8, the periodic time corresponding with any particular number being equal to $10\sqrt{I}$ seconds, where I is the moment of

inertia given in column 6 for the particular instrument in question. For example, the numbers 3200 and 283, given in columns 7 and 8 for Profs. T. and A. Gray's galvanometer, correspond with a periodic time of $10\sqrt{50}$, or about 70 seconds; while the numbers 542 and 3400, given in the same columns for the 360,000-ohm galvanometer at the Central Institution, correspond with a periodic time of only $10\sqrt{0{\cdot}01}$, or 1 second. The numbers, then, in columns 7 and 8 give a fair comparison between the various galvanometers when the time required to take an observation is unimportant.

The influence of resistance has next to be considered. If two galvanometers differ only in the gauges of wire with which they are wound, and the convolutions are similarly distributed, then the sensibilities should be proportional to the number of turns. If, again, the thickness of the insulating covering bears a constant proportion to that of the wire, then the resistance of the coils would be proportional to the square of the number of turns; so that the values of $D/r^{\frac{1}{2}}$, $S/r^{\frac{1}{2}}$ would be constants for the same pattern of instrument, where D and S are the number of scale-divisions per micro-ampere and micro-coulomb respectively, and r is the resistance of the galvanometer. The proportion in question, however, varies not only for coils of different resistances, but also in the same coil, for several different gauges are generally used in winding the various portions. To get some idea of the relation between resistance and sensibility, we have made tests on galvanometers of the same pattern but of different resistances, the same suspended system of magnets being used successively in each instrument, the controlling magnet being adjusted to produce the same periodic time in each experiment, and, when the galvanometers were reflecting ones, the scale being placed at the same distance from each of the instruments. The following were the results obtained with three astatic reflecting-galvanometers having coils of exactly the same size in each case, but of very different resistance.

Resistance of Galvanometer, in ohms.	D.	$D/r^{\frac{1}{2}}$.	$D/r^{\frac{2}{3}}$.
26	349	68·5	94·4
2457	2160	43·5	95·4
6410	3360	42	101

From this it would appear that, if the only thing altered in a galvanometer be the gauge of wire with which it is wound, the sensibility is more nearly proportional to the resistance raised to the power $\frac{2}{3}$ than to the resistance raised to the power $\frac{1}{2}$.

Another set of experiments was now made with four non-astatic galvanometers, the coils of each of which were made in the same mould, totally different, however, in shape from that employed in constructing the coils of the three reflecting-galvanometers given in the last list. When tested with the same suspended system, the controlling magnet being adjusted to produce the same periodic time in each case, the following results were obtained :—

Resistance of Galvanometer, in ohms.	M, Fraction of one degree per micro-ampere.	$M/r^{\frac{1}{2}}$.	$M/r^{\frac{2}{3}}$.
0·87	0·0149	0·01597	0·0157
4·30	0·0333	0·01604	0·0185
64·23	0·1120	0·01397	0·0246
399·25	0·2700	0·01352	0·033

In this case the sensibility appears to be more nearly proportional to $r^{\frac{1}{2}}$ than to $r^{\frac{2}{3}}$. We have, therefore, in the large table given two columns, one (No. 9) for $D/r^{\frac{1}{2}}$ and one (No. 10) for $D/r^{\frac{2}{3}}$, which represent (according as $r^{\frac{1}{2}}$ or $r^{\frac{2}{3}}$ may be regarded as the more accurate divisor to employ) the deflexion per micro-ampere on a scale whose distance is 2000 scale-divisions for a periodic time of 10 seconds that would be obtained if all the different forms of galvanometers were wound with such a gauge of wire that the resistance of

each instrument was 1 ohm. Column 11 gives the swing per micro-coulomb on the same hypothesis, but in this case we have only used $r^{\frac{1}{2}}$ as the divisor.

And whereas columns 9, 10, and 11 give the relative values for the same periodic time, columns 12 and 13 give the deflexion per micro-ampere and the swing per micro-coulomb on a scale whose distance is 2000 scale-divisions for *the same controlling-moment* that would be obtained if all the galvanometers had 1 ohm resistance—the periodic time in any case being equal to $10\sqrt{\mathrm{I}}$ seconds, I being the moment of inertia of the suspended system given in column 6.

Another question to be considered is the factor of merit in connexion with the volume of the coils. As this is important as regards the cost of high-resistance galvanometers, a separate column (5) is therefore given, showing the approximate volume in cubic centimetres occupied by the wire in each instrument, and other columns (14, 15, 16, 17) give the values obtained by dividing this quantity into the various factors of merit. The results of comparison in these latter columns are very interesting. It appears that galvanometers of the Rosenthal or Gray and the d'Arsonval types are by far the most sensitive in proportion to the volume of coil; and we believe that the best way to make a very sensitive galvanometer with a movable magnetic system is to employ several small coils, instead of one or two large ones*, and that the magnets should be horse-shoe-shaped, with the line joining the poles vertical, as in Prof. Gray's instrument, but modified so as to give smaller moment of inertia. Such a magnetic system can be made very delicately astatic, and the weakening of the magnets with time does not greatly influence the astaticism. We also conclude from our investigations that the most sensitive galvanometer of all would be one of the d'Arsonval type suitably modified. The poles of the magnet should be very close together, the coil should be very long and narrow, and no stationary iron core should be used inside the coil as in the ordinary d'Arsonval galvanometer. The sensitiveness could be further increased by employing electro-magnets instead of permanent ones to pro-

* Mr. C. V. Boys, F.R.S., in his Cantor Lectures on "Instruments for Measuring Radiant Heat" (April 1889), has also shown that galvanometers with small coils may be made very sensitive (pp. 22, 23).

duce the deflecting field, the current flowing round the electromagnet being kept constant by means of an auxiliary reflecting "set-up ammeter" producing an image on the same scale as is used for the electromagnetic d'Arsonval galvanometer. With a "set-up ammeter" the suspended system is supported by means of an almost torsionless phosphor-bronze strip, requiring many twists to be given to it to bring the spot of light on to the scale when the current, that is to be kept constant, is flowing through it. A motion, then, of the spot of light over 100 divisions of the scale corresponds perhaps to a variation of only $\frac{1}{20}$ in this current; and consequently, by means of such an instrument and a suitable adjustable resistance, a current can be kept constant to a very small fraction per cent.

It might be objected to our conclusions that, in the papers read by Prof. Threlfall before this Society, and published in the 'Philosophical Magazine' for December 1889, he comes to the conclusion that coils used in what we may shortly call the Mudford way (figs. 5, 6, and 7) give greater sensibility than when used in the Gray way (figs. 8–12). But on studying Prof. Threlfall's papers we find that, from various numerical errors, he makes his coils when used as a Gray galvanometer one ninth as sensitive as they really were, and when used as a Mudford galvanometer seventeen times as sensitive as they were; so that Prof. Threlfall's ratio of the sensibility of the Gray galvanometer to the sensibility of the same coils when used as a Mudford galvanometer requires to be multiplied by the very large number 153*.

In order to correct Prof. Thelfall's calculations, the following alterations must be made :—

Phil. Mag., Dec. 1889 : Page 465, line 11, *for* "The current was therefore $1{\cdot}26 \times 10^{-7}$ amperes," *read* "The current was therefore $1{\cdot}43 \times 10^{-8}$ amperes."

Same page, line 25, *for* "$2{\cdot}5 \times 10^{-8}$ amperes" *read* "$2{\cdot}8 \times 10^{-9}$ amperes."

Page 469, line 1: "The galvanometer having been brought to a state of sensitiveness of 5 scale-divisions for 10^{-11} amperes, the measurement of the resistance of the sample of sulphur

* These errors were also noticed by Prof. A. Gray, and referred to in an article in the Phil. Mag. for February 1890.

Type.	Description.	1. r.	2. M.	3. D.	4. S.
Thomson. One pair of coils.	Elliott's Tripod (F.T.C.) ‡	0·6	3·8	7·6	4·76*
	Tripod ,, ‡	50	102	204	128*
	Elliott's Tripod. Specially light needles †	3860	7550	15,400	9,700*
		6000	15,750	32,000	20,000*
Thomson. Two pairs of coils.	Elliott's Glass Cylinder (F.T.C.) ‡	4600	1220	2440	1520*
	Latimer Clark & Muirhead (F.T.C.) ‡	20,000	8700	10,700	6720*
	Elliott's Stock pattern †	7000	4300	7110	4450*
	,, Large Coil pattern †	12,300	7680	15,200	9550*
	,, ,, ,,	30,000	10710	21,800	13,700*
	,, Special form †	30,000	20,000	40,000	25,000*
	,, Ayrton & Perry ballistic (Japan)	20,000	1070	1430	900
	Langley's Bolometer	20	125	250	156*
Mudford's Thomson. One pair of coils.	Figure 5 (C.I.)	26	347	349	220*
	,, ,,	2,457	1375	2160	1360*
	,, ,,	6,410	2720	3360	2110*
	,, †	2086	880	1720	1080*
	Bell Magnet Ballistic †	7800	720	1350	850*
	,, ,, ,,	10,000	640	1200	755*
	Figure 5 (C.I.)	5330	.....	2280	1440*
Mudford's Thomson. Two pairs of coils.	Figure 6 (F.T.C.) ‡	700	1720	2970	1860*
	,, (C.I.)	686	1660	1600	1010*
	Variable damping Large Coils (C.I.)	9744	4200	4280	2640
	Figure 7 (C.I.)	360,000	50,000	54,200	34,000*
	,, †	350,000	15,700	31,400	19,700*
Rayleigh, Gray, or Rosenthal.	Figure 8, Non-astatic (C.I.)	44	22·9	25·4	16
	Figure 9, Astatic (C.I.)	44	25	27·7	17·4
	Non-reflecting (C.I.)	5·2	.....	9·06	5·7*
	Gray's (Proc. Roy. Soc. 1884, p. 287)	30,000	32	64	40*
	Figures 11 & 12 (C.I.)	4090	54	63	39·2
	Threlfall's (Phil. Mag. Dec. 1889, p. 460)	15680	2·90	3·74	2·35
d'Arsonval.	Jolin's High-resistance (C.I.)	750	137	260	163*
	Carpentier's ordinary	208	286	375	236*
	Jolin's Low resistance (C.I.)	3·8	39	42	26·4*
	Invariable sensibility, Large type (C.I.)	21	190	181	114*
	Nalder's type fitted with Invariable-sensibility device. Pole pieces in (C.I.)	274	.....	169	106*
	,, out ,,	274	.....	242	152*
	Carpentier milliamperemeter (C.I.)	183	.....	73	45·7*
Flat Coil or Multiplier type.	Single Needle Post-office pattern (C.I.)	800	.....	190	119*
	Coils 6 ·centim. long outside	400	.....	303	190*
	,, 2·7 ,, broad ,,	350	.....	146	91·7*
	,, 2·2 ,, high ,,	64	.....	125	78·7*
	Winding 0·6 ,, thick.	4·3	.....	14·8	9·3*
		4·3	.....	35·2	22·1*
		·87	.....	6·8	4·27*
		·87	.....	16·7	10·5*
	Detector nearly astatic †	2000	.....	490	308*
	,, partially astatic †	2000	.....	42	26·4*

5.	6.	7.	8.	9.	10.	11.	12.	13.	14.	15.	16.	17.
V.	I.	DI.	$SI^{\frac{1}{2}}$.	$D/r^{\frac{1}{2}}$.	$D/r^{\frac{2}{3}}$.	$S/r^{\frac{2}{3}}$.	$DI/r^{\frac{2}{3}}$.	$SI^{\frac{1}{3}}/r^{\frac{2}{3}}$.	$D/r^{\frac{2}{3}}V$.	$S/r^{\frac{2}{3}}V$.	$DI/r^{\frac{2}{3}}V$.	$SI^{\frac{1}{3}}/r^{\frac{2}{3}}V$.
.....	.....	.....	.....	·10	9·4	5·9						
.....	·04	81·6	25·6	28·7	42·8	26·9	17·1	5·4				
125	·01	154	970	248	566	356	5·66	35·6	4·53	2·85	0·045	0·29
125	·01	320	2000	413	986	616	9·86	61·6	7·89	4·93	0·079	0·49
.....	.....	.....	.....	36	83	52						
.....	.....	.....	.....	76	203	128						
30	0·013	92	507	85	205	129	2·68	14·7	6·8	4·3	0·089	0·049
240	0·013	198	1090	137	350	220	4·53	25·2	1·46	0·91	0·019	0·10
240	0·013	284	1560	126	353	220	4·6	25·3	1·47	0·91	0·019	0·10
250	.....	.....	.....	231	648	405	.....	.....	2·67	1·60		
.....	.....	.....	.....	10·5	27·3	17·1						
.....	.....	.....	.....	56	76	47						
72	·042	14	45	66·5	94·4	60	3·92	12·2	1·31	0·83	0·055	0·16
72	·042	91	284	43·5	95·5	60	4·00	12·3	1·33	0·83	0·055	0·16
72	·042	142	435	42	101	63	4·25	13·0	1·40	0·87	0·059	0·17
72	.....	.....	.....	37·6	81	51	.....	.....	1·12	0·71		
.....	.....	.....	.....	15·3	37·3	23·5						
.....	.....	.....	.....	12·0	30·2	19·0						
72	·042	95·5	295	31·0	74·0	46·6	3·1	8·9	1·03	0·65		
.....	.....	.....	.....	112	215	136						
88	·084	134	293	61	118	74	9·81	21·5	1·34	0·84	0·011	0·24
200	·052	222	602	43·5	108	67	5·61	15·3	0·54	0·34	0·028	0·076
236	·01	542	3400	90	324	202	3·24	20·2	1·37	0·86	0·014	0·086
.....	.....	.....	.....	53·1	190	119						
0·29	1·5	38	19·6	3·82	5·6	3·5	8·4	4·3	19·3	12·1	29·0	14·8
0·29	0·7	19·4	14·5	4·2	6·1	3·8	4·27	3·2	21	13·1	14·7	11·0
2·0	·21	1·91	2·61	4·0	4·7	2·94	·98	1·35	2·35	1·47	0·49	0·67
51	50	3200	283	0·37	1·04	0·6	52	4·6	0·02	0·012	0·10	0·09
36	17	1070	165	1·0	2·26	1·4	39	6·0	0·06	0·04	1·1	0·17
72	.....	.....	.....	·03	·078	·049	.....	.....	·00108	·00068		
2·8	3·0	7800	895	9·5	18·4	11·5	55·1	6·3	6·6	4·1	19·8	2·2
.....	.....	.....	.....	26	44·3	27·9						
0·6	5·5	230	62	21·5	25	15·5	134	36·3	42	26	225	61
0·96	13	2360	410	39·5	54	33·6	700	122	56	35	730	126
·2	1·5	254	129	10·2	17·7	11·2	27	13·7	88·5	56·0	135	68·5
.....	.....	.....	.....	14·6	25·5	16·1						
3·2	25	1820	228	5·4	9·0	5·6	225	28	2·8	1·74	70	8·7
.....	.....	.....	.....	6·7	131	82						
.....	.....	60·6	85	15·1	27·6	17·3	5·5	7·75	1·26	0·79	0·25	0·35
22	·2	29·2	41	7·8	14·0	8·8	2·82	3·94	0·64	0·40	0·13	0·18
.....	.....	25	35	15·6	23·8	14·9	4·75	6·65	1·08	0·68	0·21	0·30
.....	.....	2·96	4·1	7·1	8·25	5·18	1·65	2·28	0·37	0·23	0·07	0·10
.....	.....	7·04	9·9	17·0	19·6	12·3	3·92	5·5	0·85	0·56	0·18	0·25
.....	.....	1·4	1·9	7·3	7·2	4·53	1·49	2·02	0·32	0·21	0·07	0·09
.....	.....	3·3	4·7	18	17·9	11·1	3·5	5·0	0·82	0·50	0·15	0·23
.....	.....	.....	.....	1·1	23·4	14·7						
.....	.....	.....	.....	·09	2·0	12·6						

in question became a tolerably easy matter." On examining, however, the results obtained in the actual measurements with the sulphur (p. 471), we find that the sensibility of the galvanometer was not as above stated, 500,000 divisions per micro-ampere, but 34,200. And, as his scale was 3 metres away from the galvanometer, and his scale-divisions 1 millim., this number 34,200 reduced to our standard becomes 22,800*.

EXPLANATION OF TABLE.

r = resistance in ohms.

M = millimetres deflexion per micro-ampere when the period is 10 seconds, and the scale is placed as in the actual use of the instrument.

D = deflexion in scale-divisions per micro-ampere when the period is 10 seconds, and the scale-distance is equal to 2000 scale-divisions.

S = swing produced per micro-coulomb under the same conditions as in last.

V = volume occupied by the convolutions of wire, in cubic centimetres approximately.

I = moment of inertia of the suspended system, in C.G.S. units approximately.

DI = deflexion in scale-divisions (scale-distance equal to 2000 scale divisions) per micro-ampere for constant controlling moments, and for a periodic time equal to $10\sqrt{\text{I}}$ seconds.

$\text{SI}^{\frac{1}{2}}$ = swing per micro-coulomb under the same conditions as in last.

Columns 9, 10, and 11 give the deflexion per micro-ampere, and the swing per micro-coulomb, when the period is 10 seconds and the resistance of each instrument one ohm.

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* Prof. Threlfall says, in a letter published in the Phil. Mag. for June 1890, in reply to these criticisms, that his galvanometer was not in a state of maximum sensibility when used for these experiments on sulphur. Granting this, and looking at the observations given on page 472 of the Phil. Mag., vol. xxviii., from which the constant of the galvanometer was determined, we find that there was considerable vagueness of the zero, amounting in some cases to more than 4 per cent. of the double deflexion, as well as a gradual change in the position of the zero from 105 to 78. If, then, the control were diminished to about $\frac{1}{17}$ of the value it then had, and which would be necessary to give the galvanometer the sensibility claimed for it, the vagueness of the zero would probably become too serious for the instrument to be used for any but the very roughest measurements.

Columns 12 and 13 give the deflexion per micro-ampere and the swing per micro-coulomb, for the same controlling moment, and for the resistance of each instrument equal to one ohm.

Columns 14 and 15 give the deflexion per micro-ampere and the swing per micro-coulomb per cubic centimetre of coil, when the period is 10 seconds and the resistance of each galvanometer one ohm.

Columns 16 and 17 give the deflexion per micro-ampere and the swing per micro-coulomb per cubic centimetre of coil, for the same controlling moment, and the resistance of each galvanometer equal to one ohm. Periodic time of any instrument is $10\sqrt{\mathrm{I}}$ seconds.

C.I., Central Institution.

F.T.C., Finsbury Technical College.

† The constants of these instruments have been kindly furnished by the makers.

‡ These instruments were tested after being in use for some time and without their needles being remagnetized.

* These numbers are calculated from those given in column 3.

VIII. *Self-Induction of Galvanometer-Coils.*

We have already seen that the sensibilities of two precisely similar galvanometers, which differ only in resistance, cannot easily be predicted from their resistances, since, in consequence of the varying thickness of the insulating covering of the wire, there is no simple connexion between resistance and the number of convolutions of wire occupying a given space. It therefore occurred to us to consider whether, since the self-induction of a coil of a given size and shape depends mainly on the square of the number of convolutions, and but little on the thickness of the wire or of the insulating covering, the square root of the self-induction would be proportional to the magnetic constant of the coil. The following table gives the results of the tests on the three astatic reflecting-galvanometers already referred to :—

Galvanometer.	Resistance, in ohms. r.	Self-Induction, in secohms. L.	Time Constant, in seconds.	D.
A	26	0·052	0·002	349
B	2457	1·75	0·00071	2160
C	6410	4·72	0·00074	3360

D, as before, standing for the number of scale-divisions of deflexion produced by the same current with the same distance of the scale, the same system of mirror and magnets being used in each case, and the controlling magnet adjusted to give the same periodic time of oscillation. The following gives the ratio of the values of D, of the square roots of the self-inductions, of the square roots of the resistances, and of the resistances to the power $\frac{2}{3}$:—

$$\frac{D_B}{D_A} = 6{\cdot}190\,; \quad \sqrt{\frac{L_B}{L_A}} = 5{\cdot}801\,; \quad \sqrt{\frac{r_B}{r_A}} = 9{\cdot}721\,; \quad \left(\frac{r_B}{r_A}\right)^{\frac{2}{3}} = 6{\cdot}167$$

$$\frac{D_C}{D_B} = 1{\cdot}555\,; \quad \sqrt{\frac{L_C}{L_B}} = 1{\cdot}643\,; \quad \sqrt{\frac{r_C}{r_B}} = 1{\cdot}615\,; \quad \left(\frac{r_C}{r_B}\right)^{\frac{2}{3}} = 1{\cdot}468$$

$$\frac{D_C}{D_A} = 9{\cdot}627\,; \quad \sqrt{\frac{L_C}{L_A}} = 9{\cdot}526\,; \quad \sqrt{\frac{r_C}{r_A}} = 15{\cdot}70\,; \quad \left(\frac{r_C}{r_A}\right)^{\frac{2}{3}} = 9{\cdot}051$$

From the preceding list it appears that the ratio of the square roots of the self-inductions gives the ratio of the galvanometer-constants of coils, not only much more accurately than the ratio of the square roots of the resistances, but, on the whole, even more accurately than the ratio of the resistances to the power $\frac{2}{3}$.

In view, then, of the importance of knowing the self-induction of galvanometer-coils, we give the results of some measurements made by means of the secohmmeter :—

	Resistance, in ohms.	Self-Induction, in secohms.	Time-Constant, in seconds.
Reflecting-Galvanometer with One Pair of Coils.			
Front coil	3007	3·75	0·00125
Back coil	2830	3·16	0·00112
Front and back coils, joined up as in actual use	5837	9·6	0·00164
Mutual induction of the two coils in position	.....	1·35	
Reflecting-Galvanometer with Variable Damping.			
Four coils in series, but separated so as have little Mutual Induction	9680	12·48	0·00129
Reflecting High-Resistance Galvanometer (fig. 7).			
Top front coil	105,600	40·72	0·000386
Top back coil	101,200	39·50	0·000390
Bottom front coil	90,000	27·51	0·000306
Front top and back coils joined up in position	207,000	109·8	0·000531
Reflecting Ballistic Galvanometer (fig. 11).			
One Coil only	1003	0·166	0·000166
Flat Single-Coil Galvanometer. (Nobili type.)			
The one Coil	64	0·0324	0·000506

From the above it appears that a coil of the ordinary size and shape used in Thomson galvanometers has a coefficient roughly equal, numerically, to one thousandth of its resistance in ohms. If the coils of the instrument are in series and placed near each other, as in actual use, the coefficient is increased by about half its former amount, owing to mutual induction; so that, for Thomson galvanometers whose resistances vary between 1000 and 10,000 ohms, the time-constant is about 0·0015 second. With very high-resistance instruments the time-constant falls to 0·0005 second, in consequence of the increasing importance of the insulating covering of the wires.

IX. *Effect of Time and Use on Galvanometer-Needles.*

The following give the values of D, the deflexion in divisions of a scale, placed at a distance from the galvanometer equal to a length occupied by 2000 scale-divisions, produced by one micro-ampere, the controlling magnet being adjusted to give a periodic time of oscillation equal to 10 seconds.

Double-Coil Astatic Reflecting-Galvanometer. Resistance, 686 ohms.

	D.
Needles magnetized and tested the same day	1600
After about two years' use	366
Needles remagnetized and then used for about one year	826

Single-Coil Astatic Reflecting-Galvanometer. Resistance, 26 ohms.

As obtained from the makers	198
After about three years' use	170
Needles remagnetized and tested the same day	243

Double-Coil Astatic Reflecting-Galvanometer; Variable Damping. Resistance, 9744 ohms.

As obtained from the makers	4200
After about three years' use	3000
Needles remagnetized and tested the same day	4280

Rosenthal Non-astatic Reflecting Micro-Galvanometer. Resistance, 44 ohms. (Figs. 8 and 9.)

As obtained from the makers	25·4
After four years' use	5·16

The needles of the first and last of these four galvanometers have lost much more magnetism than have the needles of the second and third galvanometers. As regards the Rosenthal instrument it might have been expected, in view of the powerful nature of its magnetic needle (see fig. 9) and of the fact that the current only acts on small projections of this magnet, that the magnetism would not be much diminished by use. But it has to be remembered that, whereas with the ordinary reflecting-galvanometers the zero-position of the needles is that in which the lines of force due to the coil are at right

angles to the axes of the magnetic needles, and therefore the needles are not in a position to have their magnetism much altered by the passage of a current, unless that current be very strong, the suspended magnet of the Rosenthal galvanometer will tend to have its magnetism diminished by every current sent through the instrument in such a direction as to push the poles of the suspended magnet out of the coil, since the lines of force of the two coils will pass along the axis of the suspended magnet. Care not having been taken to send the current round the Rosenthal galvanometer, so as to always suck the ends of the magnet into the coils, is probably the explanation of the weakening of its suspended magnets during four years in the ratio of 5 to 1; whereas the falling-off in the sensibility of the first of the four galvanometers in the preceding list in the ratio of 4·3 to 1 during a shorter time is probably due to inferior steel having been used in the construction of its magnetic needles.

In view of the great difference which exists between various specimens of magnet-steel, it is most important that only the very best magnet-steel should be employed in the construction of galvanometers which are intended to have a high sensibility, since the factor of merit of the instrument depends largely on the goodness of the magnetic needles.

Mr. C. V. Boys, F.R.S., thought that the factor of merit of galvanometers should not be given in scale-divisions per micro-ampere under the condition of constant controlling moment. This gave too great an advantage to instruments of the Gray or Rosenthal type. Great sensibility could be obtained by diminishing the moment of inertia of the suspended parts, the practical limit being determined by the trouble due to the silk fibre. Spider lines, when used in place of silk fibres, gave better results. It was possible, by using a good suspending arrangement, to use needles $\frac{1}{8}''$ long, and a period of twenty seconds, and to gain a sensibility far greater than those indicated in the paper. Ballistic galvanometers should be made with needles as light as possible. The method proposed of winding the central part of the coil in the opposite sense to the rest would probably not be good, owing to the unevenness of the field produced. The conclusion come

to by the authors that D'Arsonval galvanometers of great sensibility should be made with small coils placed in a very strong field, was one he had himself come to, but had finally abandoned, owing to difficulties caused by diamagnetism in the copper and to excessive damping due to Foucault currents.

Mr. Swinburne thought that the factor of merit of a galvanometer should be determined differently, according as it was to be used for the measurement of current, or quantity, or for null methods merely. He saw no great advantage in making practical instruments proportional. The name D'Arsonval should be dropped, as the instrument denoted by it was invented by Varley years ago. He would like to know the relative sensibility of the telephone and the Lippmann galvanometer.

Prof. Fitzgerald stated that Lord Rayleigh had shown that the microscope-method of observing angular deflexions was as sensitive as the ordinary method of mirror and scale, even when only the mirror was used as a pointer, so that if a pointer were attached it would be far more sensitive. The drawback, however, was that it was impossible to distinguish, with the microscope, between lateral displacements of the needles and the angular motion whose measurement was required. To get over this error it was necessary to read both ends of the pointer; but this was hard to do.

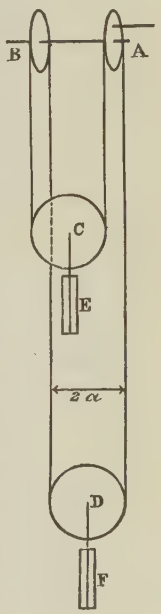
LVI. *On* Huygens's *Gearing in illustration of the Induction of Electric Currents.* *By* Lord Rayleigh, *Sec. R.S., Professor of Natural Philosophy in the Royal Institution**.

As a mechanical model of the electric machinery at work in the induction of currents Maxwell employed differential gearing; and an apparatus on this principle, designed by him, is in use at the Cavendish Laboratory. Wishing to show something similar in a recent course of lectures, and not having differential gearing at my disposal, I designed more than one combination of pulleys, the action of which should be analogous to that of electric currents. These eventually resolved themselves into Huygens's gearing, invented, I

* Read May 16, 1890.

believe, in connexion with the winding of clocks. As this apparatus is easier to understand than differential gearing, and the parts of which it is composed are more likely to be useful for general purposes in a laboratory, I have thought that it might be worth while to give a description, accompanied by an explanation of the mode of action.

Two similar pulleys, A, B, turn upon a piece of round steel fixed horizontally*. Over these is hung an endless cord, and the two bights carry similar pendent pulleys, C, D, from which again hang weights, E, F. The weight of the cord being negligible, the system is devoid of potential energy; that is, it will balance, whatever may be the vertical distance between C and D.



Since either pulley A, B may turn independently of the other, the system is capable of two independent motions. If A, B turn in the same direction and with the same velocity, one of the pendent pulleys C, D rises, and the other falls. If, on the other hand, the motions of A, B are equal and opposite, the axes of the pendent pulleys and the attached weights remain at rest.

In the electrical analogy the rotatory velocity of A corresponds to a current in a primary circuit, that of B to a current in a secondary. If when all is at rest the rotation of A be suddenly started, by force applied at the handle or otherwise, the inertia of the masses, E, F, opposes their sudden movement, and the consequence is that the pulley B turns *backwards, i. e.* in the opposite direction to the rotation imposed upon A. This is the current induced in a secondary circuit when an electromotive force begins to act in the primary. In like manner, if A having been for some time in uniform movement suddenly stops, B enters into motion in the direction of the former movement of A. This is the secondary current on the break of the current in the primary circuit.

* Light wooden laths, variously coloured and revolving with the pulleys, render the movements evident at a distance.

It must be borne in mind that in the absence of friction there is nothing to correspond with electrical resistance, so that the conductors must be looked upon as perfect. The frictions which actually enter do not follow the same laws as electrical resistances, and only very imperfectly represent them. However, the frictions which oppose the rotations of A and B have a general effect of the right sort; but the rotations of C and D, corresponding to dielectric machinery, should be as free as possible.

The effect of a condenser, to which the terminals of one of the circuits is joined, would be represented by a spiral spring (as in a watch) attached to the corresponding pulley, the stiffness of the spring being inversely as the capacity of the condenser. The absence of the spring, or (which comes to the same thing) the indefinite decrease of its stiffness, corresponds to infinite electrical capacity, or to a simply closed circuit.

The equations which express the mechanical properties of the system are readily found, and are precisely the same as those applicable in the electrical problem. Since the potential energy vanishes, everything turns upon the expression for the kinetic energy. If x and y denote the circumferential velocities in the same direction of the pulleys A, B where the cord is in contact with them, $\frac{1}{2}(x+y)$ is the vertical velocity of the pendent pulleys. Also $\frac{1}{2}(x-y)$ is the circumferential linear velocity of C, D, due to rotation, at the place where the cord engages. If the diameter be here $2a$, the angular velocity is $(x-y)/2a$. Thus, if M be the total mass of each pendent pulley and attachment, Mk^2 the moment of inertia of the revolving parts, the whole kinetic energy corresponding to each is

$$\tfrac{1}{2}M\left\{\frac{(x+y)^2}{4}+\frac{k^2}{a^2}\frac{(x-y)^2}{4}\right\}.$$

For the energy of the whole system we should have the double of this, and, if it were necessary to include them, terms proportional to x^2 and y^2 to represent the energy of the fixed pulleys. The reaction between the pulleys A, B depends upon the presence of a term xy in the expression of the energy. We see that this would disappear if $k^2=a^2$; as would happen if the whole mass of the pendent pulleys and attachments

were concentrated in the circles where the cord runs. The case discussed above, as analogous to electric currents, occurs when $k^2 < a^2$, a condition that will be satisfied, even without non-rotating attachments, if the cord run near the circumference of the rotating pulleys. The opposite state of things, in which $k^2 > a^2$, would be realized by carrying out masses beyond the groove, and thus increasing the rotatory in comparison with the translatory inertia. In this case the mutual action between A and B is reversed. If when all is at rest A be suddenly started, B moves forward in the *same* direction. Otherwise C and D would have to rotate, and this in their character of fly-wheels they oppose.

Generally, if L, N be the coefficients of self-induction, and M mutual induction, we have (constant factors being omitted)

$$L = N = a^2 + k^2,$$
$$M = a^2 - k^2.$$

In order to imitate the case of two circuits coiled together in close proximity throughout, we must have in the mechanical model $k^2 = 0$; that is, the rotatory inertia of the pendent pulleys must be negligible in comparison with the translatory inertia. Also the energy of the fixed pulleys, not included in the above expressions, must be negligible. If these conditions be satisfied, a sudden rotation imposed upon A generates an *equal* and opposite motion in B.

LVII. *On the Diurnal Variation of the Magnet at Kew.* By W. G. Robson *and* S. W. J. Smith, *of the Normal School of Science and Royal School of Mines**.

The multiplication of magnetic observatories makes it extremely desirable that the methods of reducing the observations should be as similar as possible. It is, however, practically impossible that all observatories should determine the diurnal variations of the elements by the elaborate method in use at Greenwich, which, in the case of any one element, necessitates the measurement of the hourly ordinates

* Read June 6, 1890.

of the curves at 24 points on all but a very few days of the year under discussion.

Dr. Wild (Rep. Brit. Assoc. 1885, p. 78) advocates the determination of the mean diurnal range from measurements on the comparatively small number of days of very small disturbance, and points out that the inclusion of disturbed days will affect the form of the curve of diurnal range unless the disturbances themselves occur indifferently at all hours of the day and night.

Mr. Whipple, the Superintendent of the Kew Observatory, made a comparison of the methods of Wild and Sabine, and that in use at Greenwich for the years 1870–71–72 (Rep. Brit. Assoc. 1886, p. 71).

His general conclusion was that the difference between the Kew results, as treated by the two methods of Sabine and Wild, was less than the difference between the Greenwich and Kew results in either case.

The range of the differences is given below :—

Mean of three years 1870–72.

	Range of Differences.
Kew (Sabine and Wild)	−0′·3 to +0′·4
Greenwich and Kew (Sabine)...	−0′·6 to +0′·6
Greenwich and Kew (Wild) ...	−0′·9 to +0′·7

It would have been interesting if similar comparisons could have been carried out for more recent years ; but the fact that the Kew magnetic year has not been identical with that used at Greenwich has made such a comparison impossible. The Kew magnetic year has hitherto been from October to October, to admit of the observations being included in the Annual Report made by the Committee to the Royal Society. We understand that in future the civil year will be adopted.

Besides this, some of the Kew results have been obtained not by measuring the hourly ordinates, but by drawing free-hand a mean curve among the superposed traces of "quiet day" curves.

Table I.

Year.	1.	2.	3.	4.	5.	6.	7.	8.	9.	10.	11.	Noon. 12.	1. 13.	2. 14.	3. 15.	4. 16.	5. 17.	6. 18.	7. 19.	8. 20.	9. 21.	10. 22.	11. 23.	Mid. 24.	
1883.	−1·42	−1·34	−1·42	−1·58	−1·88	−2·18	−2·69	−3·01	−2·51	−0·68	+1·93	+4·29	+5·52	+5·38	+4·15	+2·57	+1·32	+0·32	−0·29	−0·75	−1·24	−1·39	−1·55	−1·53	Greenwich.
	−0·72	−0·77	−0·98	−1·26	−1·80	−2·29	−3·11	−3·77	−3·32	−1·28	+1·50	+3·89	+5·24	+5·06	+3·61	+2·17	+0·90	+0·29	−0·09	−0·35	−0·53	−0·69	−0·92	−0·82	Kew.
1886.	−1·61	−1·46	−1·42	−1·38	−1·57	−1·79	−2·02	−2·07	−1·52	−0·03	+2·17	+4·26	+5·21	+4·92	+3·79	+2·39	+1·14	+0·12	−0·64	−1·18	−1·60	−1·93	−1·99	−1·86	Greenwich.
	−1·29	−1·17	−1·24	−1·57	−1·95	−2·23	−2·59	−2·71	−2·18	−0·47	+1·86	+3·84	+4·87	+4·43	+3·20	+1·91	+0·91	+0·23	−0·04	−0·38	−0·68	−0·81	−0·97	−1·13	Kew.
1887.	−1·47	−1·18	−1·05	−1·15	−1·42	−1·62	−1·88	−1·96	−1·35	+0·20	+2·33	+4·09	+4·91	+4·45	+3·17	+1·83	+0·74	−0·05	−0·71	−1·07	−1·47	−1·85	−1·84	−1·74	Greenwich.
	−1·00	−0·94	−1·04	−1·25	−1·60	−2·06	−2·47	−2·76	−2·01	−0·25	+1·95	+3·93	+4·66	+4·20	+2·69	+1·39	+0·57	−0·04	−0·34	−0·48	−0·57	−0·80	−0·96	−0·84	Kew.

Table II.

Year.	1.	2.	3.	4.	5.	6.	7.	8.	9.	10.	11.	Noon. 12.	1. 13.	2. 14.	3. 15.	4. 16.	5. 17.	6. 18.	7. 19.	8. 20.	9. 21.	10. 22.	11. 23.	Mid. 24.	
1870 ...	−0·9	−0·8	−0·7	−0·2	+0·2	+0·2	+0·5	+0·4	+0·6	+0·6	+0·4	+0·1	+0·5	+0·5	+0·4	+0·6	+0·6	+0·2	0·0	−0·1	−0·4	−0·8	−1·1	−1·0	Greenwich and Kew
1871 ...	−0·8	−0·6	−0·6	−0·4	+0·2	+0·7	+0·9	+0·7	+0·7	+0·7	+0·2	+0·5	+0·5	+0·7	+0·8	+0·6	+0·3	+0·1	−0·1	−0·2	−0·9	−0·7	−1·1	−0·9	(Whipple)
1872 ...	−0·9	−0·7	−0·6	−0·5	−0·1	+0·3	+0·5	+0·8	+0·8	+1·0	+0·6	+0·5	+0·7	+0·6	+0·8	+0·5	+0·5	+0·1	−0·1	−0·7	−0·7	−0·9	−1·1	−0·8	Greenwich
1883 ...	−0·7	−0·6	−0·4	−0·3	−0·1	+0·1	+0·4	+0·8	+0·8	+0·6	+0·4	+0·4	+0·3	+0·3	+0·5	+0·4	+0·4	0·0	−0·2	−0·4	−0·7	−0·7	−0·6	−0·7	and Kew
1886 ...	−0·3	−0·3	−0·2	+0·2	+0·4	+0·4	+0·6	+0·6	+0·7	+0·4	+0·3	+0·4	+0·3	+0·5	+0·6	+0·5	+0·2	−0·1	−0·6	−0·8	−0·9	−1·1	−1·0	−0·7	(R. and S.)
1887 ...	−0·5	−0·2	0·0	+0·1	+0·2	+0·4	+0·6	+0·8	+0·7	+0·5	+0·4	+0·2	+0·3	+0·3	+0·5	+0·4	+0·2	0·0	−0·4	−0·6	−0·9	−1·1	−0·9	−0·9	
Mean...	−0·7	−0·5	−0·4	−0·2	+0·1	+0·4	+0·6	+0·7	+0·7	+0·6	+0·4	+0·4	+0·4	+0·5	+0·6	+0·5	+0·4	+0·1	−0·2	−0·5	−0·8	−0·9	−1·0	−0·8	

It was therefore determined to make some further comparison of the different methods of reduction, and at the request of Professor Rücker we have determined the diurnal variation of the declination at Kew for three recent civil years.

The years selected were 1883, a year of maximum sun-spots, 1886, and 1887. The mean diurnal range for each year was found by Wild's method—the hourly ordinates for about five quiet days in each month being measured.

The results we obtained have been compared with the corresponding Greenwich results, as published by the Royal Observatory (see Table I.). Thus for the total range for each year at Kew and Greenwich we have the following :—

Total Ranges.

Year.	Kew.	Greenwich.
1883	9′·0	8′·5
1886	7′·6	7′·3
1887	7′·4	6′·9

It is noteworthy that the total range at Kew determined from quiet days is greater than that at Greenwich determined from all days except those of great disturbance. If we may argue from so small a comparison this indicates that storms on the whole diminish instead of increasing the diurnal excursions of the needle. The total range is also apparently decreasing yearly.

The range of differences of the corresponding mean hourly values is :—

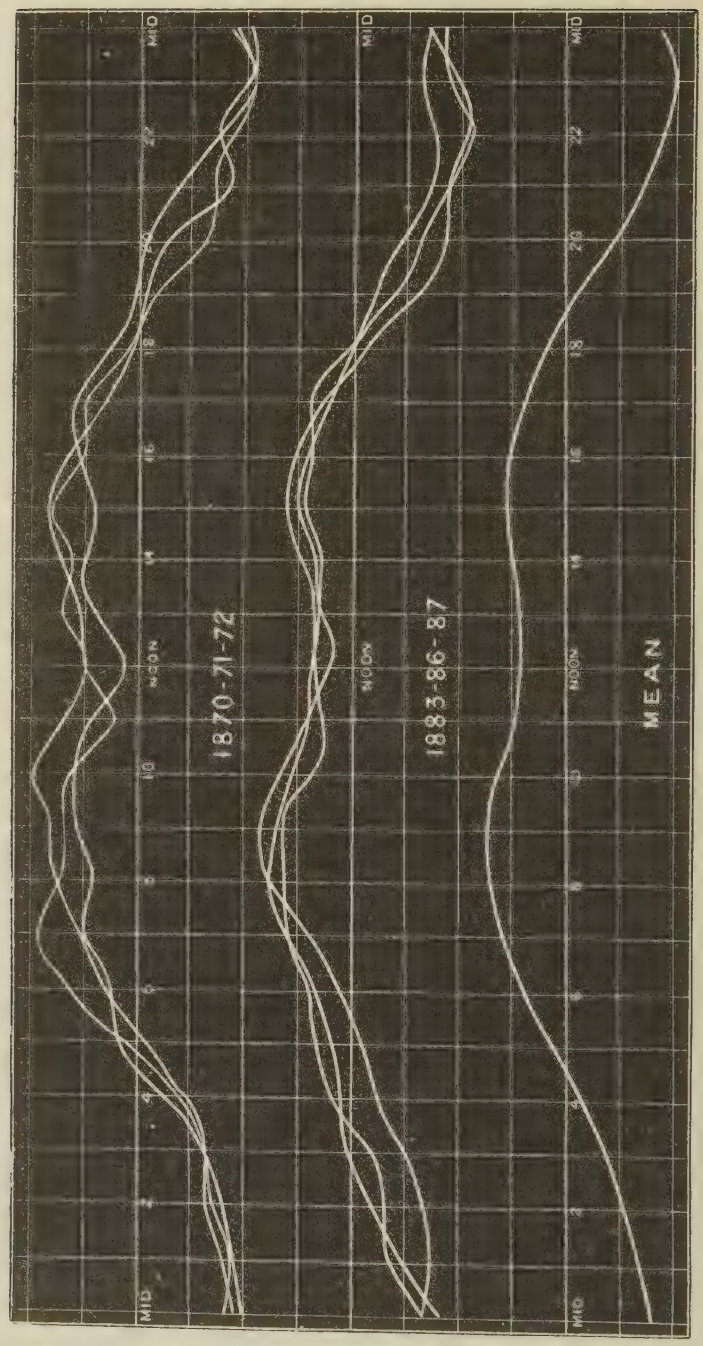
	Range of Differences.	Year.
Greenwich and Kew ...	−0′·7 to +0′·8	1883
,, ,,	−1′·1 to +0′·7	1886
,, ,,	−1′·1 to +0′·8	1887

There is thus in 1883 a difference of 1′·5 in a range of 9′—the percentage being 16·7. For 1886 the difference is 1′·8 in a range of 7′·6—or 23·7 per cent.; while for 1887 the difference is 1′·9 in a range of 7′·4, or 25·7 per cent.

Table II. shows the differences of the hourly readings for the three years just determined, together with the differences for the years 1870, 1871, and 1872, as determined by Mr. Whipple (Rep. Brit. Assoc. *loc. cit.*). The mean of those hourly differences for the six years has been taken (see Table II.). It is noticeable that there is considerable uniformity in the nature of the differences between the mean hourly readings for Greenwich and Kew respectively. Thus, for each year, the Greenwich mean curve falls below that obtained from the Kew magnetograph at about 7 P.M., and remains below it until about 4 A.M., from which hour it is above the Kew curve until it crosses it at about 7 P.M. The minimum ordinate for each curve for each year occurs at about 8 A.M., and the maximum at 1 P.M. The declination has its mean value between 10 A.M. and 11 A.M., and again at between 6 P.M. and 7 P.M. Precisely the same features are to be noticed in the tables given by Mr. Whipple for the years 1870, 1871, and 1872 (Rep. Brit. Assoc. 1886, p. 74); so that it may fairly be assumed that the general nature of the difference between the Greenwich and Kew mean curves from 1870 to 1887 is the same.

The general similarity of the differences is shown clearly by the accompanying curves—the abscissæ of which represent hours, while the ordinates give the mean differences between the Greenwich and Kew readings at these hours. The first set are plotted from the data given in Mr. Whipple's paper. The second set are from the data which we have obtained for the years 1883, 1886, and 1887. The lowest represents the mean of the six curves.

The individual readings do not differ from the corresponding mean to an extent greater than 0′·4. It would therefore seem possible, knowing one set of values for any particular year—Greenwich or Kew—to determine the other set, correct to within four tenths of a minute. As the Kew results are published earlier than the Greenwich ones, it would be possible to calculate approximately the latter from the former.


MID
2
4
6
8
10
NOON
14
16
18
20
22
MID
1870-71-72
1883-86-87
MEAN

Referring to the reduction of results, Prof. Rücker said hat the Stonyhurst observers and Prof. Mascart were willing o adopt Wild's method ; Falmouth, he hoped, would follow uit, and Greenwich had been asked to publish their results n both ways.

Mr. Whipple said that before recording-instruments were vailable, and the numbers were obtained from separate expe-iments, the labour involved was considerable, and a single arge disturbance or magnetic storm might vitiate the result f a whole year's work. Methods were therefore adopted to o eliminate these disturbances ; of these, that used by Sabine nay be particularly mentioned. Although declination-records ave now been obtained for a considerable number of years, he cause of the variations still remains unknown. They do ot seem to be dependent on temperature or on astronomical acts. He considered it valuable to obtain magnetic data rom different parts of the earth, but comparisons were only ossible when all are published on the same plan. This, he oped, would result from the efforts of Professors Rücker and Adams. When this is accomplished, the observations on magnetic force will need treatment ; the work will be labo-ious, and the aid of volunteers like Messrs. Robson and Smith would be of great service.

Prof. W. G. Adams said he was glad to see the satisfactory nature of the work which had just been brought before the Society. Uusually the mass of figures to be dealt with was so large that the mere reduction was a great undertaking. If, however, the difference between results obtained by the Greenwich and Wild's method was not more than 0′·4, it may be possible to make out the causes of the variations from observations reduced on Wild's plan. He himself would put more faith in horizontal-force observations, and wished they could be worked out by some ready method. He hoped the one adopted in America, of obtaining mean curves by photo-graphy, may prove satisfactory.

Prof. Perry asked if a machine could not be made to do the work.

Mr. Whipple said such machines had been used by the Meteorological Office, but they were so elaborate and expen-

sive that clerical work was just as cheap. The method o photographing mean curves had been tried at Kew, but was open to the objection that accidental disturbances such a those produced by the movement of iron in the vicinity an the approach of cabs &c. were not eliminated.

Mr. Boys, referring to the use of integrators, said that fo a harmonic analyser his disk-cylinder pattern was preferab to the ball disk-cylinder integrators of J. Thomson, for it i much cheaper and has less inertia.

The President said the movement initiated by Prof. Rücke would be of great service if it resulted in the numbers ob tained at the various magnetic observatories being publishe in the same way. It was a great advantage to have suc men, who were not permanently attached to an observatory to take up the subjects and suggest improvements. The heads of such institutions were usually too much employed in making the necessary reductions to have time for devising improved methods. In his opinion greater freedom should be allowed to the chiefs of observatories, for it should be borne in mind that the object of observations is not to produce volumes of figures but to increase our knowledge.

Referring to the reduction of observations, he thought the voluntary services of senior physical students should be more generally accepted, and to this end he suggested that properly recommended persons should be allowed to spend some time in observatories as honorary assistants. This would be of great use to the students themselves and an advantage to the observatories, for the reduction of observations could then be expedited. As regards the accidental disturbances referred to by Mr. Whipple, he contended that regulations should be adopted to render them impossible.

VIII. *The Effect of Change of Temperature on the* Villari *Critical Point of Iron**. *By* Herbert Tomlinson, *F.R.S.*†

[Plates XII. & XIII.]

This paper must be regarded as a continuation of one previously communicated to the Physical Society‡ on the Villari Critical Points of Nickel and Iron. As before, only the temporary magnetization was studied, and the mode of conducting the observations was also the same with the addition that the effect of loading was tested not only at the temperature of the room but also at various temperatures extending to 285° C. The temperature was ascertained from the resistance of a coil of platinum wire wound double and embracing the same part of the iron wire as the secondary coil; the temperature-coefficients of the platinum coil had previously been very carefully determined§. The iron wire used was 1 millim. in diameter; it had been thoroughly annealed, and the loads employed were not sufficient, even at the highest temperature, to produce any very sensible permanent extension. As a preliminary the wire was heated to 300° C., tested with all the loads and magnetizing forces to be presently mentioned, and again tested when cooled to the temperature of the room. This treatment was repeated several times until constant results were obtained at both temperatures; the observations recorded in figs. I.–VI. were then made. In these figures the abscissæ represent the number of kilogrammes in the load and the ordinates the percentage alteration of permeability at the temperature indicated on each curve, + signifying increase and − decrease of permeability. The Villari critical point is the value of the magnetizing force for which loading does not affect the permeability and may be ascertained by noting the point at which each curve cuts the line of loads; it varies with the value

* Read June 6, 1890.

† The author expresses his thanks to the Elizabeth Thompson Science Fund for aid in this research.

‡ *Ante*, p. 367.

§ For further information concerning the platinum coil and the primary and secondary coils, see Proc. Phys. Soc. vol. ix. p. 181.

of the load and with the temperature. In some instances the curves cut the line of loads *twice*, and there are consequently *two* loads which for the magnetizing forces pertaining to the curves have no effect on the temporary magnetization. In the following table is given the load (or loads if there are two) which for each particular magnetizing force and temperature is without effect on the permeability.

TABLE I.

Magnetizing force in C.G.S. units.	Load for which the permeability is the same as for the unloaded wire. Temp. 12° C.	Ditto. Temp. 76° C.	Ditto. Temp. 167° C.	Ditto. Temp. 244° C.	Ditto. Temp. 285° C.
2·84	4·6	4·8	5·1 and 12·0	5·8 and 10·1	
3·70	2·5	3·2	3·6	4·2 and 11·5	4·7 and 9·9
4·83	1·8	2·5	2·7	not tested.	3·1 and 12·3

It will be seen from Table I. and from figs. I.–IV. that the magnetizing forces 2·84, 3·70, 4·83, and of course all intermediate forces, are Villari critical values. If we regard only the *first point* where each curve cuts the line of loads, we see that at all temperatures the Villari value *increases* as the load *decreases* and, for a given load, increases with the temperature. If, on the contrary, we consider the *second point* of cutting the line of load, we find the critical value *increases both with the load and with the temperature**.

It will be seen from fig. I. that as the temperature rises the two points of cutting the load-line creep nearer and nearer together until for a temperature a little below 285° C. they coincide, and the curve osculates the load-line; whilst for 285° C. and for higher temperatures the magnetizing force 2·84 ceases to be a Villari point for any load. When the

* The effect of change of temperature on the critical value is therefore in the same direction for both points of cutting the line of loads, *i. e.* in both cases the Villari value is increased by rise of temperature.

magnetizing force reaches the value 7·69 (fig. IV.) both points of cutting the load-line vanish, at any rate when the loading is not carried beyond 12 kilos*, and for magnetizing forces beyond 7·69 (figs. V.–VI.) Villari critical values become impossible.

In all cases rise of temperature increases the alteration of the permeability in the positive direction and diminishes it in the negative direction ; but the diminution in the latter case is greater than the increment in the former, so that the summits of the two bends in the curves become more and more nearly on the same level.

The percentage alteration of permeability was calculated as follows :—

If P_0 and P_n represent the permeability with no load and a load of n kilogrammes respectively, the percentage alteration is

$$\frac{100\,(P_n - P_0)}{P_0} \text{ or } \frac{100\,(P_0 - P_n)}{P_0},$$

according as the loading produces increase or decrease of permeability. Table II. gives the values of P_0 for the various magnetizing forces.

Table II.

Magnetizing force in C.G.S. units.	Temporary permeability in C.G.S. units of the unloaded wire at 12° C.	Ditto at 76° C.	Ditto at 167° C.	Ditto at 244° C.	Ditto at 285° C.
2·84	301	306	338	353	385
3·70	325	340	383	385	405
4·83	347	366	413	.....	461
7·69	478	483	548	.....	557
10·40	528	573	638	.....	675
15·32	522	.....	.....	.....	617

* The second point could, however, be probably reached by taking the load to higher values.

LIX. *Notes on Secondary Batteries.* *By* J. H. Gladstone, *Ph.D., F.R.S., and* W. Hibbert, *F.I.C.**

Eight years ago, one of us, in conjunction with the late Mr. Tribe, published a series of papers on the Chemistry of Secondary Batteries †. Since that time accumulators have undergone many improvements in detail, but they still present some unsolved problems of a more or less subtle character. In this paper we give an account of some experiments undertaken to test points which have arisen in the last few years.

Action of Sodium Sulphate.

In May 1889 Mr. Preece delivered a lecture at the Society of Arts dealing generally with his experience of Secondary Batteries Among the topics treated was the influence of the composition of the electrolyte on the working of the cell, and it was stated that the addition of sodium sulphate to the acid solution was of very considerable advantage. Mr. Preece determined the proportions of acid and sulphate which gave the best result in a year's working, and found that a comparatively small quantity of the salt was sufficient to produce the effect.

We have determined the percentage composition of the liquid from a sample kindly furnished by Mr. Preece, and find it to be as follows:—

Sodium sulphate (Na_2SO_4) .	0·81
Sulphuric acid (H_2SO_4) . .	29·39
Water	69·80
	100·00

The specific gravity at ordinary temperatures is about 1·23 ‡. When used in secondary batteries instead of the usual acid,

* Read June 20, 1890.

† Nature, 1882 and 1883, vols. xxv. and xxvi.; afterwards published by Macmillan.

‡ In making this liquid, it would be best to add together 68 parts (by weight) of water, 30 of strong sulphuric acid, and 2 parts of crystallized sodium sulphate ($Na_2SO_4 . 10H_2O$).

it is stated that there is less buckling of the plates and little or no scaling. The chief benefit, however, is stated to be that the sodium salt diminishes the chance of objectionable sulphating in the cell, and where such sulphating occurs, enables it to be removed by a charging current much more readily than when sulphuric acid alone is employed.

Mr. Preece did not explain the beneficial action of the sodium sulphate, but in the discussion following the lecture Sir F. Abel threw out the idea that it was "probable that a double salt of sulphate of lead and sodium was formed which was soluble, whereas the sulphate of lead was comparatively insoluble."

We have made some experiments to test the value of this suggestion. It may be well, however, to note that lead sulphate is so nearly insoluble in dilute sulphuric acid that comparatively large errors are easily possible.

We took advantage of the greater solubility of lead sulphate in strong acid. Measured volumes of this were added to water, the precipitate formed allowed to settle, and the lead salt remaining in solution determined by precipitation as sulphide.

The mean results are :—1 part of lead sulphate dissolves in 105,000 parts of the acid mixed with water, in 90,000 parts of a similarly diluted acid containing 5 per cent. sodium sulphate, and in 107,000 parts of Mr. Preece's solution.

As these numbers deal with such minute quantities we lay no stress on their absolute value. Their relative values are more useful, and evidently the addition of the sodium salt has not materially increased the solubility of lead sulphate.

This conclusion was confirmed by experiments of another type. To equal quantities of a dilute acid and a similar acid containing sodium sulphate we added a few drops of strong acid saturated with lead sulphate. Precipitates were formed in both cases, but it was possible so to adjust the number of drops that the precipitate did not appear for an appreciable time. We could therefore see in which it appeared first. In many experiments and many variations of procedure, we nearly always obtained earlier precipitation in liquids containing sodium or magnesium sulphates than in those consisting simply of dilute acid. We judge therefore that lead sulphate is not more soluble in these liquids, and that the explanation

of the action of sodium sulphate must be sought in some other direction.

Mr. Swinburne has suggested that, as lead sulphate is dissolved by sodium hydrate, the action may be due to that hydrate being produced by electrolysis of the sodium sulphate. Experiment showed that in a simple solution of neutral sulphate of soda, a paste made of equal parts of red lead and sulphate of lead was more readily reduced than in dilute sulphuric acid. It was also easy to detect sodium hydrate on the cathode by red litmus-paper. But this last result was no longer obtained when the sodium-sulphate solution contained sulphuric acid in the proportion of 1 volume to 10. The acid present interferes with such a simple test, and we sought for information on the point by determining the ratio of acid to sulphate of soda in the pores of the spongy lead, and comparing it with the ratio of the same substances in the body of the liquid. Preliminary experiments indicated the following as the best course to pursue.

A strip of spongy lead was used as cathode in a liquid somewhat similar to that analysed above. The ratio was $\frac{H_2SO_4}{Na_2SO_4}=30$. A current was passed for three hours, falling slowly from about 2 amperes to 1·5. The strip was then taken out and placed in 120 cubic centim. of distilled water and allowed to diffuse for 40 minutes. It was then placed in fresh distilled water for 18 hours. The forty minutes' diffusate gave a ratio of $\frac{H_2SO_4}{Na_2SO_4}=25$, only a little lower than that shown by the original liquid. The 18 hours diffusate gave a ratio $\frac{H_2SO_4}{Na_2SO_4}=11{\cdot}8$. Evidently, then, the portions of the liquid which diffused last were much richer in sodium salt than the earlier ones.

This would show that a large proportion of sulphate of sodium is formed in close contact with the lead sulphate, and we have already seen that this is favourable to the reduction of that body.

But there is another consideration. It is well known that the spongy lead formed on one of the plates in a secondary battery is to some extent acted on by the liquid.

It occurred to us that this direct chemical action between lead and sulphuric acid might be influenced by sodium sulphate. We therefore placed a quantity of red lead on a sheet of lead and reduced it electrolytically. The sheet was then cut into small pieces and divided into two equal quantities of about 20 grammes each. One half was put into a flask containing 1 volume of acid to 6 of water; the other half was put into a similar acid with the addition of 5 per cent. of sodium sulphate. Hydrogen gas was evolved from both flasks at the ordinary temperature. The following Table shows the results:—

Time, in days.	Hydrogen collected, in cubic centimetres.			
	From Acid.		From Acid containing Na_2SO_4.	
	Per diem.	Total.	Per diem.	Total.
1	160	160	116	116
2	125	285	91	207
3	120	405	87	294
7	75	695	54	512

It is evident that the sodium sulphate exercised a protecting influence on the lead.

In another experiment with a solution containing as small a proportion of sodium sulphate as that used by Mr. Preece, the protective influence was still observable, though much less marked.

These results support the idea that part of the effect produced by sodium sulphate lies in its power to diminish the local actions arising between the acid and the different parts of the plates.

It is not unlikely that this chemical action has something to do with the difficulty experienced in electrolytically reducing pure lead sulphate when that substance unmixed with other compounds is simply immersed in dilute acid; for at the moment of incipient decomposition the lead must be very finely divided, with its affinities in their most active state.

Sub-sulphate of Lead.

In a paper read before the Royal Society*, Dr. Frankland doubts whether the lead sulphate formed and decomposed in the cell is the ordinary white sulphate known to chemists, and has studied the action of dilute acid on litharge and on minium. From litharge he obtained a buff-coloured compound, for the composition of which he gives the formula $(SO_3)_3(PbO)_5$. From red lead he obtained a brownish-red compound, $S_2Pb_3O_{10}$. Dr. Frankland thinks it highly probable that one or other of these salts takes part in the electrolytic processes of the storage battery. He remarks, however, very truly that the working conditions in a battery are very different from those of his experiments.

With reference to these results we have made some observations.

Buff-coloured body from Litharge.—The formula $(SO_3)_3(PbO)_5$ can be written $2PbO + 3PbSO_4$. In other words, the substance may either be a true sub-sulphate, or a mixture of the ordinary sulphate and oxide of lead. When attempting to prepare some of this compound, preliminary results suggested the following experiment.

Known quantities of litharge and sulphuric acid were digested together in a flask. From time to time the flasks were shaken vigorously and a small quantity of the clear liquid subtracted. The strength of the acid was carefully determined; and from this, together with the known volume of acid, it was easy to find how much acid had combined with the lead oxide. The curves given below, which are drawn through the actual points of observation, show the course of the action for 28 days, the ordinates giving the acid absorbed, and the abscissæ the time.

In Exp. I., 33·41 grams of litharge and 150 cubic centim. of 1 to 5 acid were used. This liquid contained 43·38 grams of sulphuric acid

In Exp. II., 39·73 grams of litharge and 150 cubic centim. of 1 to 6 acid $=36{\cdot}37$ grams H_2SO_4 were used.

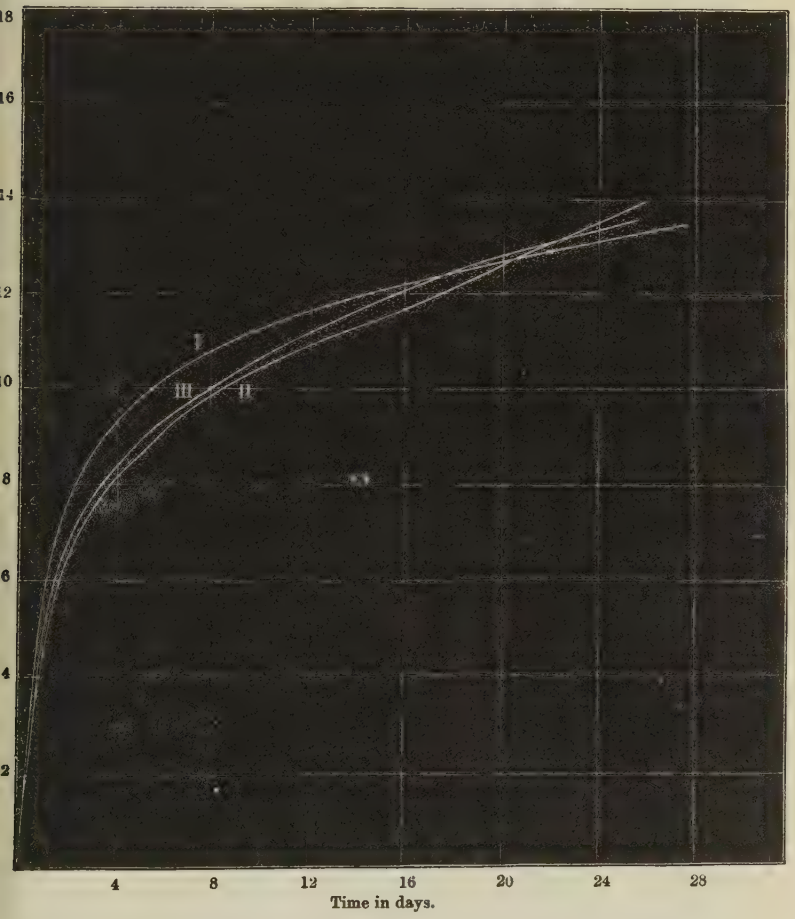
In Exp. III., 39·92 grams of litharge and 150 cubic centim. of 1 to 10 acid $=25$ grams H_2SO_4 were used.

* Proc. Roy. Soc. 1888.

The acid required for total conversion of the litharge into sulphate would be,

In Exp. I. = 14·7 grams.
In Exp. II. = 17·45 „
In Exp. III. = 17·54 „

Of course only three fifths of these quantities would have been required to form the compound $(SO_3)_3(PbO)_5$.



The salt gradually became white in colour. It is evident that in each case the action was slow and progressive, without any sudden change of rate to mark the formation of a sub-salt.

At a later date we stopped the experiments and analysed the substances produced in Exps. I. and II.

In I. we found 31·26 per cent. of SO_4, which indicates the presence of 98·6 per cent. $PbSO_4$ and 1·4 per cent. of PbO.

In II. we found 31·67 per cent. of SO_4, which indicates 99·5 per cent. $PbSO_4$ and 0·5 per cent. of PbO.

The general outcome is clearly in favour of the idea that the action is a continuous one, which will only finish when the whole of the litharge is converted into sulphate. As the experiment proceeds there is an increasing proportion of insoluble sulphate present, which no doubts acts as a protective coat for the unchanged litharge, and thus renders the transformation less rapid. We found in fact that the action was greatly hastened if we continually broke up the substance in a mortar.

We see therefore no reason to doubt the view put forth as far back as January 1882, that the substance produced in the voltaic reaction is ordinary sulphate of lead, a substance which is easily oxidated by means of the current, and is reduced without serious difficulty, especially when broken up by better conducting bodies.

High Initial Electromotive Force.

During the prosecution of the experiments described in the preceding pages our attention was again drawn to the high electromotive force which, as is well known, is found in a secondary battery for a short time after the charging current has been stopped. So far as we know, the explanations of this curious phenomenon which have been hitherto given are not looked upon as satisfactory. These explanations depend on the presence and action of electrolytic gases, which are either supposed to be occluded by the electrodes, or else to form secondary products differing from the ordinary elements in much the same way as ozone differs from oxygen.

We thought it probable that the chief cause of the high E.M.F. might be found in the great inequalities in the strength of acid produced by the charging current. There is no doubt that the acid becomes much stronger in the immediate vicinity of the peroxide plate. The strong and weak acid thus produced at the positive and negative plates respectively, being

contained in spongy masses of material, require some time to diffuse, and the normal E.M.F. of 2 volts cannot be attained until diffusion has produced uniformity in the strength of the acid. We put this idea to experimental test.

Most of the observations were made on a small experimental cell in which were:—

(1) a negative plate consisting of a strip of lead (1 inch × 3) covered with spongy lead;

(2) a positive plate made of similar lead covered with peroxide;

(3) an electrolyte of dilute acid containing 18·5 per cent. H_2SO_4 (= 1 volume of acid to about 9 of water).

We also provided a small porous earthenware jar filled with stronger acid, into which we could introduce the peroxide plate when desired.

In the first series of experiments the plates were charged in the uniform electrolyte by a current of nearly half an ampere, and after breaking the battery-circuit, observations were taken of the E.M.F. by the condenser method.

Time, after stopping the charging current.	Electromotive force, in volts.			
	Exp. I.	Exp. II.	Exp. III.	Exp. IV.
At once	2·58	.....	.....	2·58
5 seconds	.....	.....	.....	2·28
1 minute	2·16	2·18	2·17	2·14
2 minutes	2·13	2·17	2·16	2·12
5 „	2·06	2·10	2·10	2·06
10 „	2·01	2·04	2·02	2·05
15 „	2·00	2·01	2·01	

Peroxide plate now put into the jar of stronger acid.

Time after change of acid round peroxide.	Acid 34 per cent.*	Acid 34 per cent.*	Acid 34 per cent.*	Acid 58 per cent.*
½ minute	2·05 volts.	2·06 volts.	2·05 volts.	2·16 volts.
1 „	2·07 „	2·09 „	2·09 „	2·20 „
2 „	2·07 „	2·12 „	2·12 „	2·21 „
4 „	2·09 „	.....	.....	2·24 „
5 „	2·10 „	2·13 „	2·13 „	

From this Table it is evident that the rate of fall in the various experiments is fairly uniform, and also that an

* These strengths refer to the acid in the porous jar. That in the general body of the cell remained as before—18·5 per cent.

increase in the strength of the acid round the positive plate (peroxide) is able to give a large increase in the E.M.F. of the cell. We further see that the greater portion of the fall in E.M.F. takes place in the first few seconds after stopping the charging current. This we should expect if the action depends on differences of density in acid, for these differences will be greatest and diffusion most marked in the early stages. In the following set of experiments the spongy lead stood in 18·5 per cent. acid, while the porous jar containing the peroxide plate was filled with acid whose strength in Exps. I. and II. was 34 per cent., and in Exp. III. 58 per cent. The cell thus arranged was charged as before, and after stopping the charging current observations were taken of the E.M.F.

Time, after stopping the charging current.	Electromotive force, in volts.		
	Exp. I.	Exp. II.	Exp. III.
At once	2·61	.....	2·61
5 seconds	.....	.....	2·44
1 minute	2·22	2·24	2·33
2 minutes	2·21	2·23	2·31
5 „	2·18	2·21	2·28
10 „	2·17	2·20	2·28
15 „	2·14	2·16	2·27
25 „	.....	.....	2·27
30 „	.....	.....	2·25

From this Table it appears that the rate of fall is very much slower than before. This is especially marked in Exp. III., where the strength of the acid in the jar is nearer the possible maximum producible in the interstices of the spongy materials.

It may be worth notice, that the value of the E.M.F. to which the cells finally *rise* in Exps. I., II., and III. of the first series is very nearly the same as that to which they *fall* in the first two experiments of the second series. If the rise is due, as we think, to the diffusion inwards of the stronger acid, some such agreement might be expected.

We therefore regard the abnormal E.M.F. as due to the nequality of acid strength, and its gradual disappearance as due to equalization of strength produced by diffusion.

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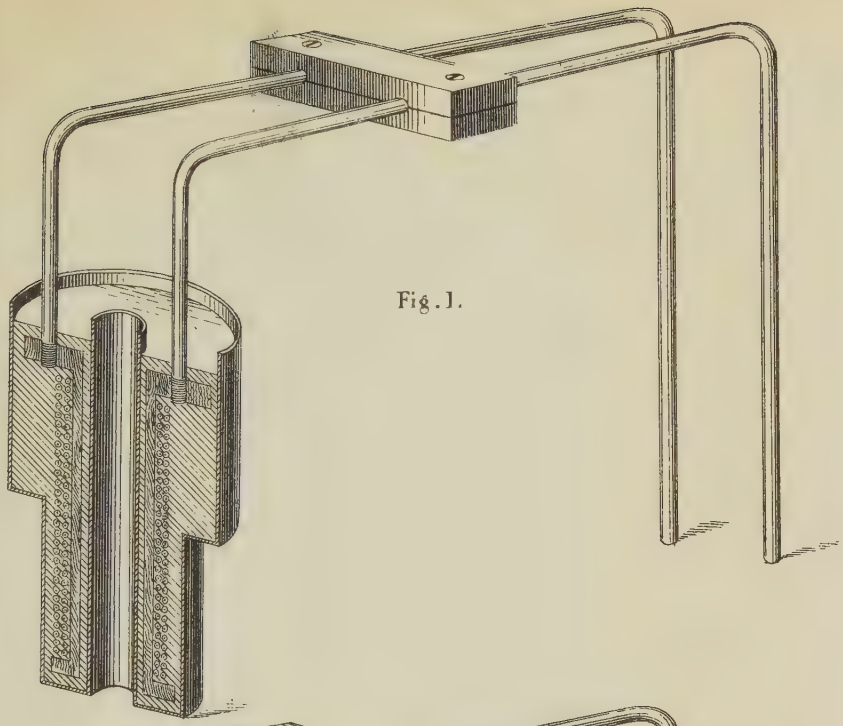

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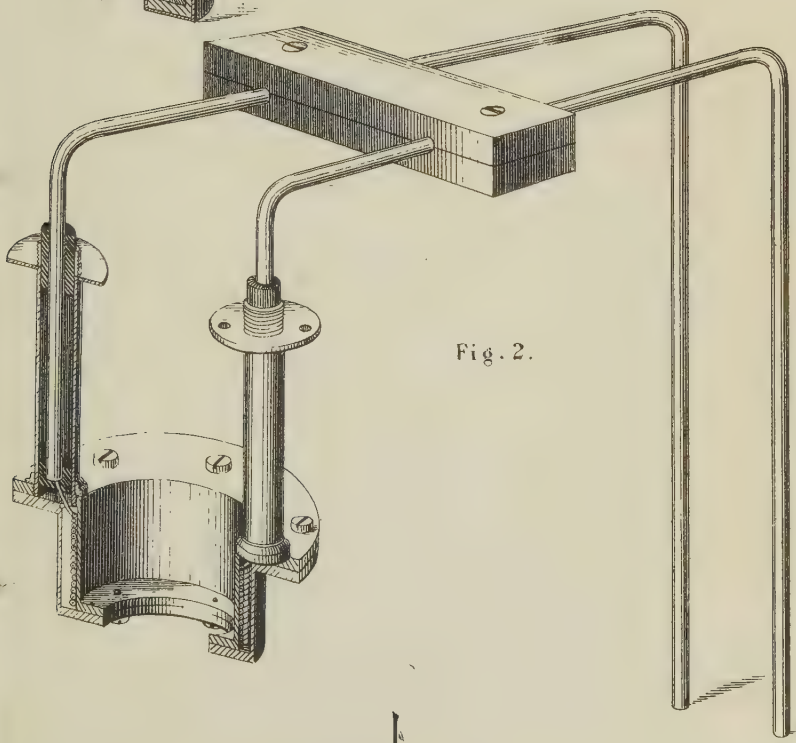

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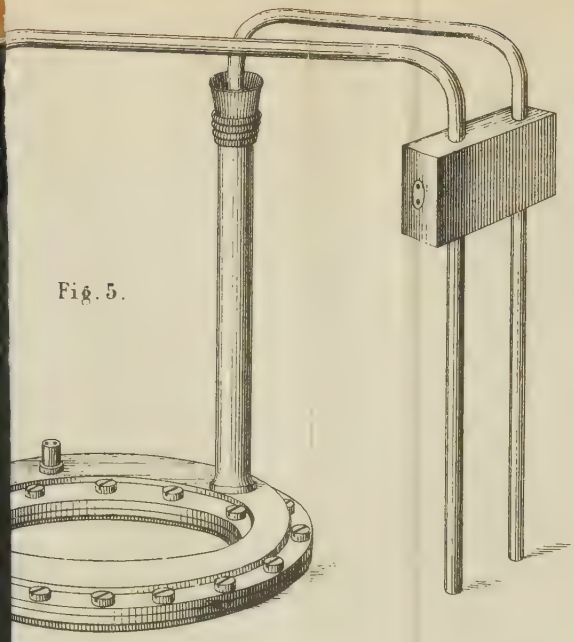

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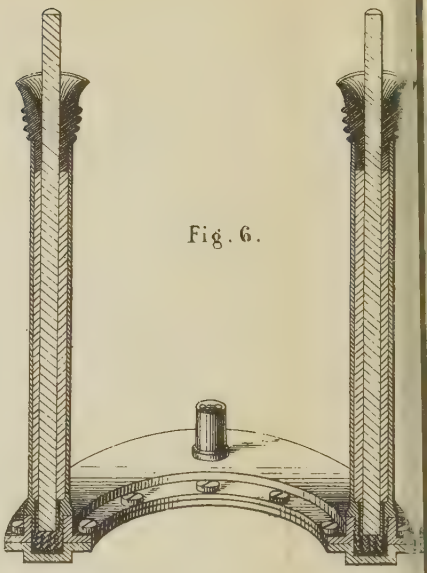

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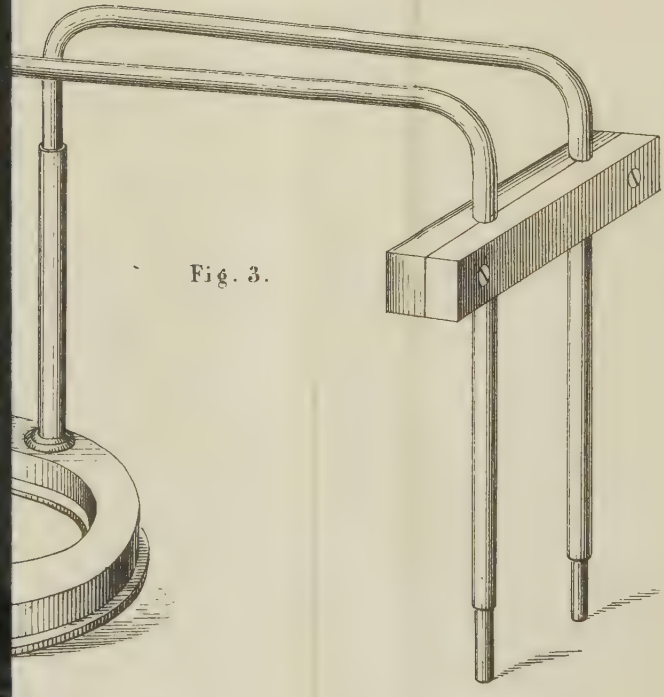

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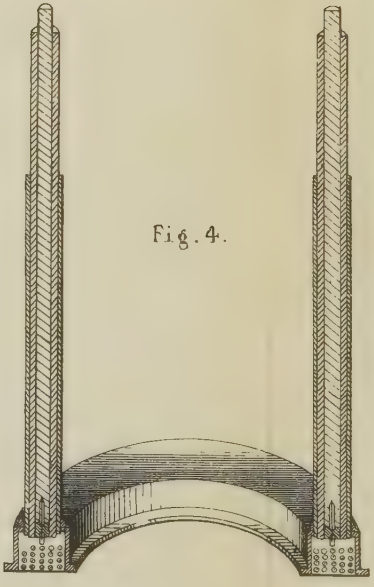

Fig. 4.

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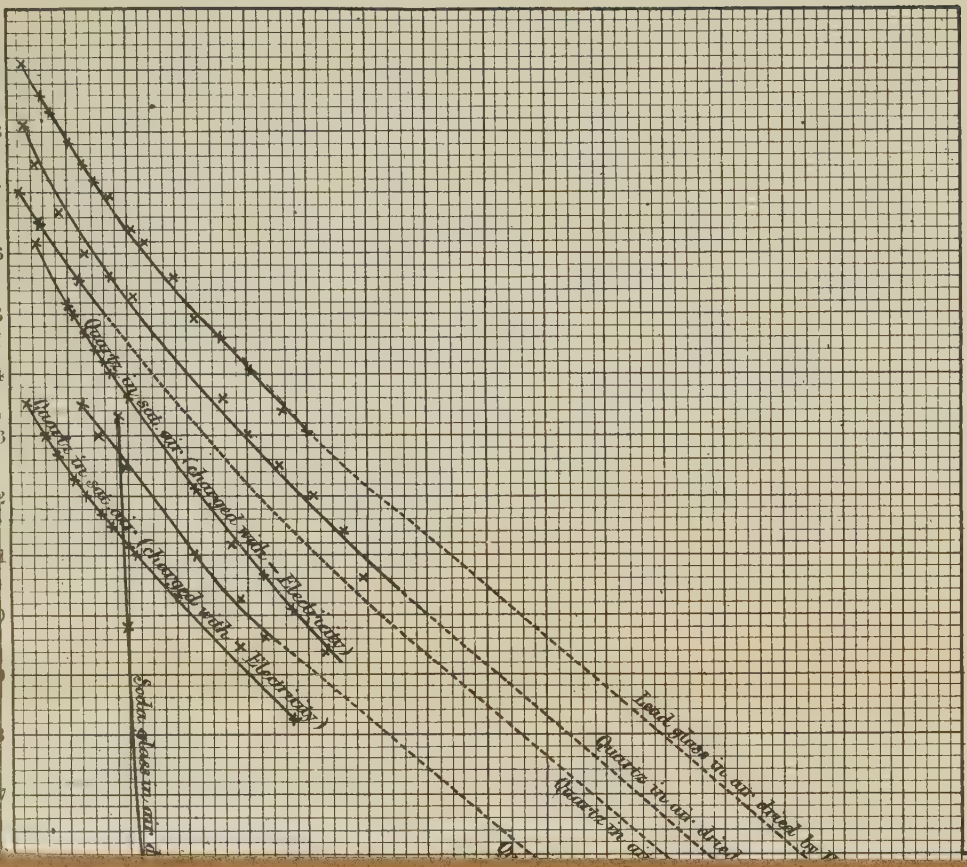
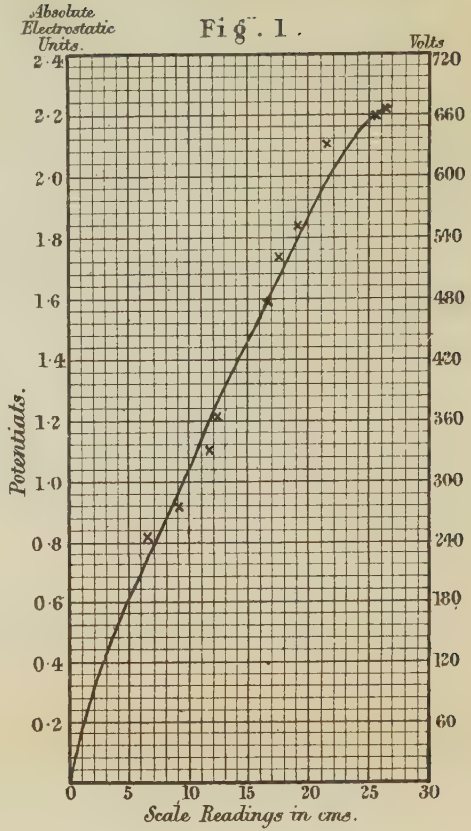

Quartz in air dried by P2O5
Quartz in air dried
Quartz in air
Quartz in air (charged with − Electricity)
Quartz in air (charged with + Electricity)
Soda glass in air

Fig. 1.


Absolute Electrostatic Units.
Volts
Potentials.
2·4
2·2
2·0
1·8
1·6
1·4
1·2
1·0
0·8
0·6
0·4
0·2
720
660
600
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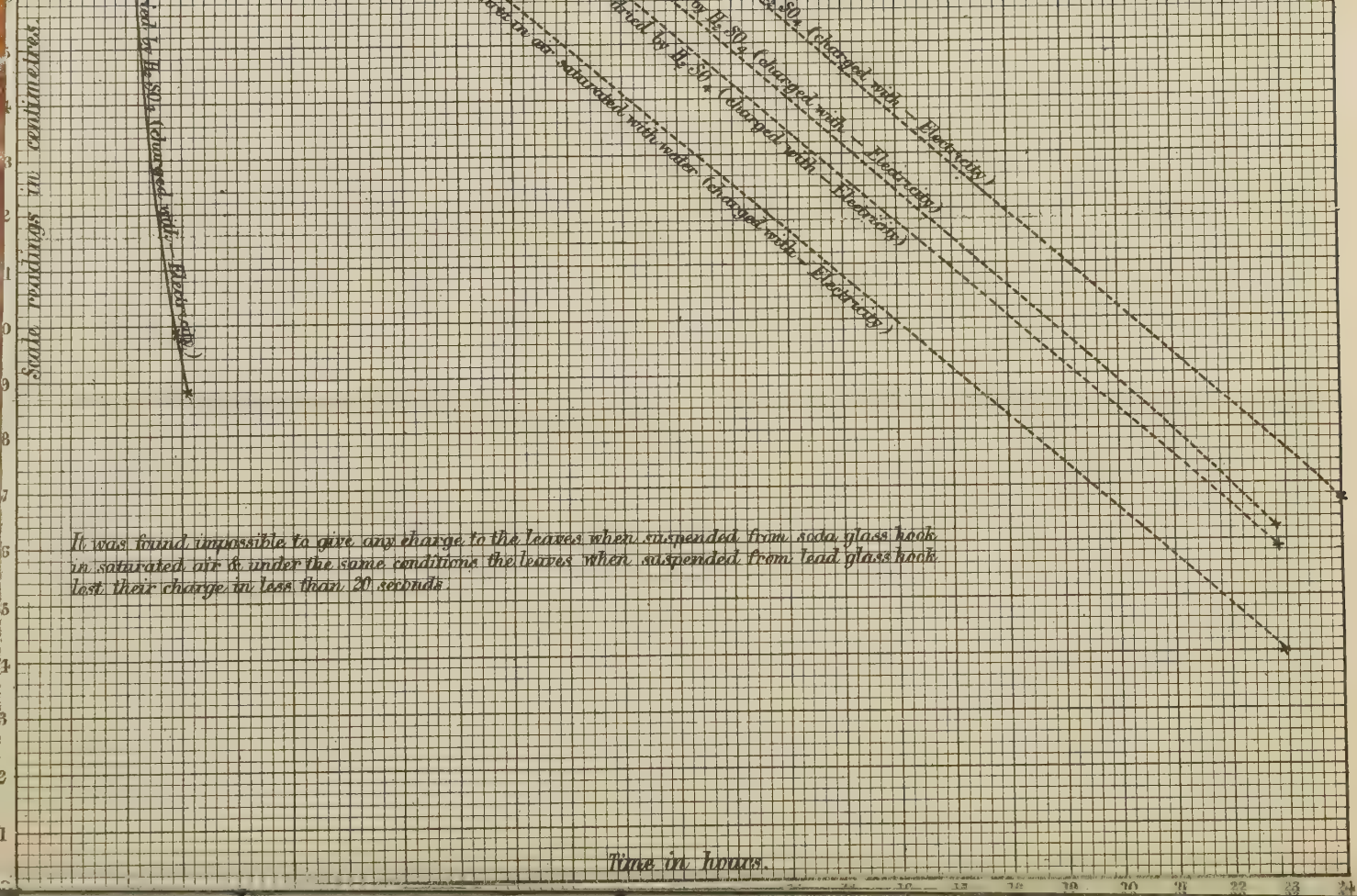
Scale Readings in cms.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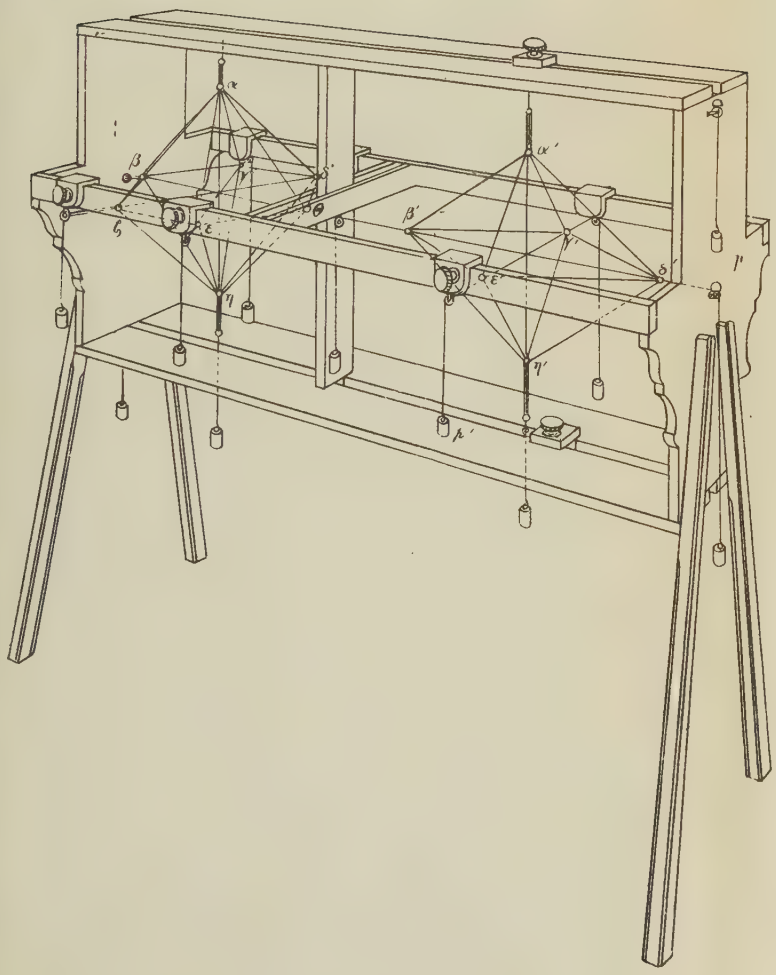

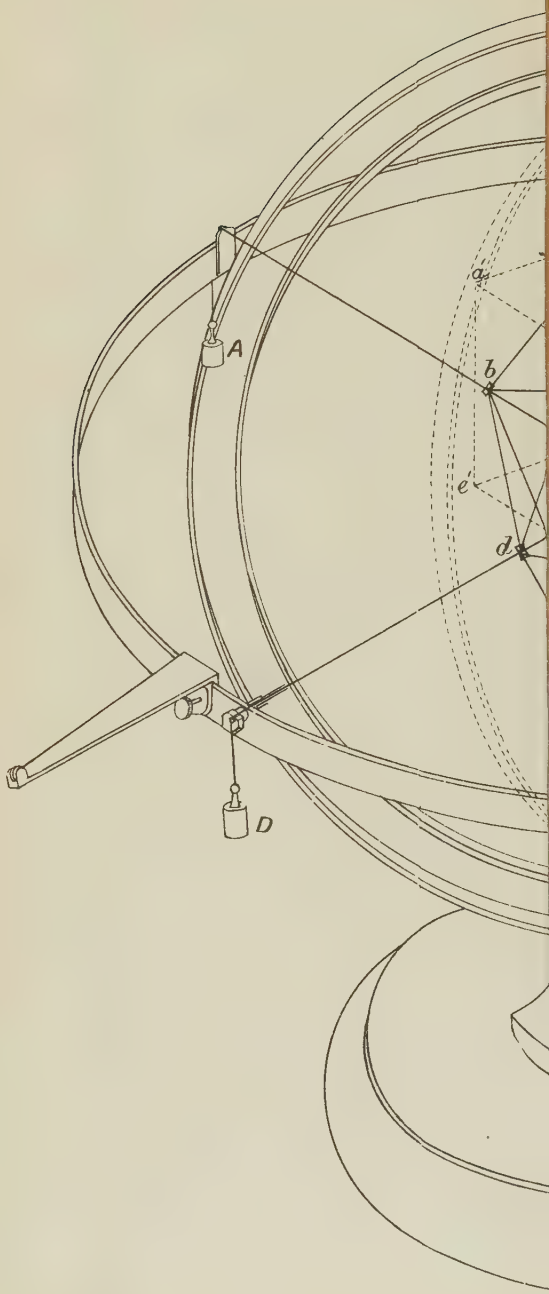

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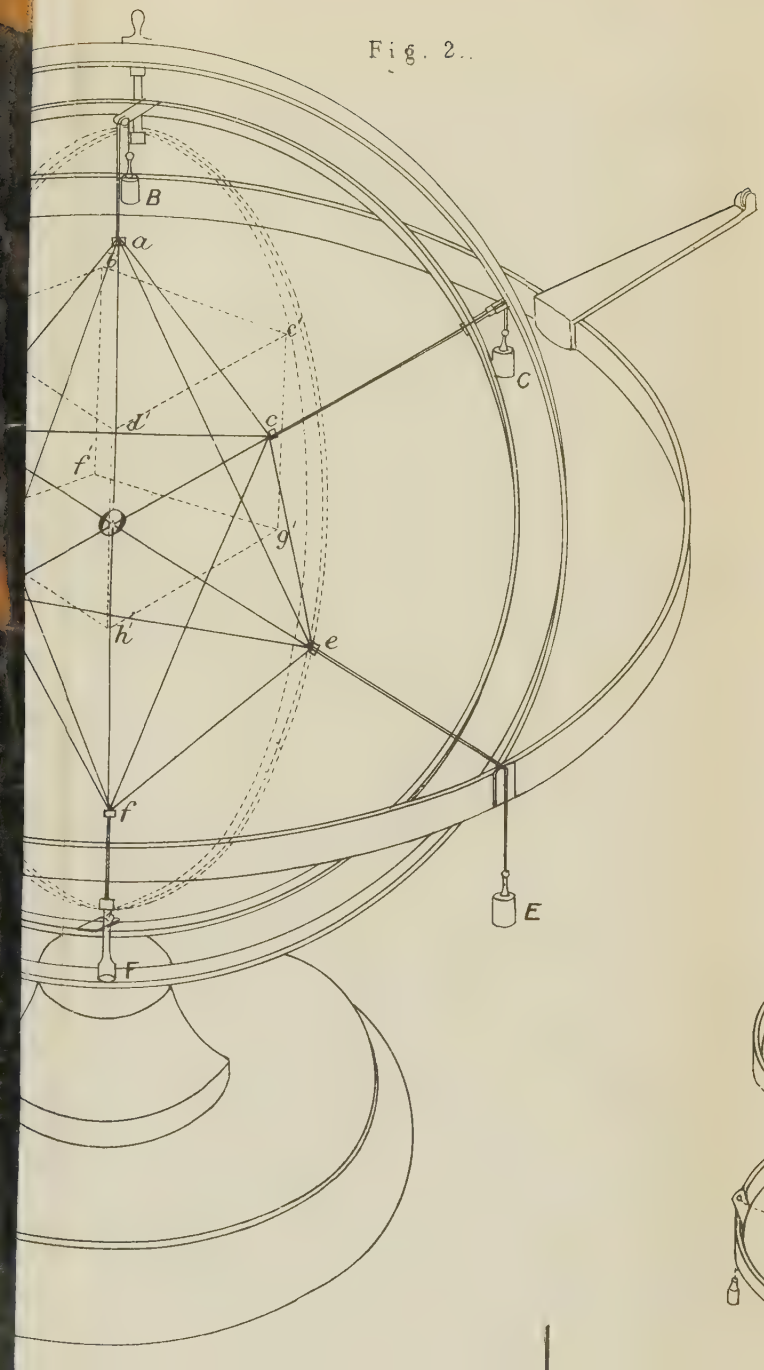

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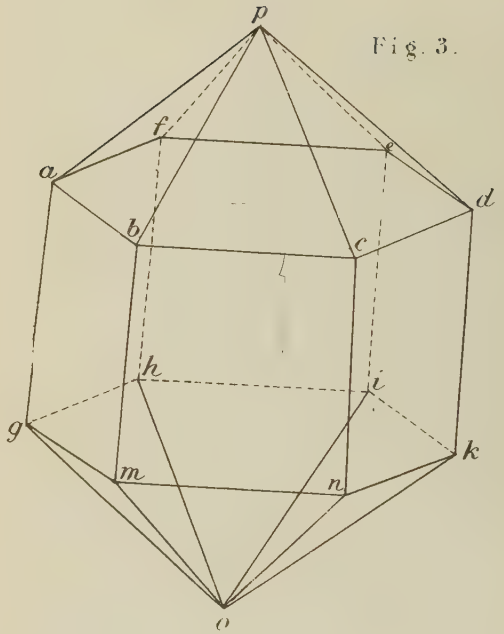

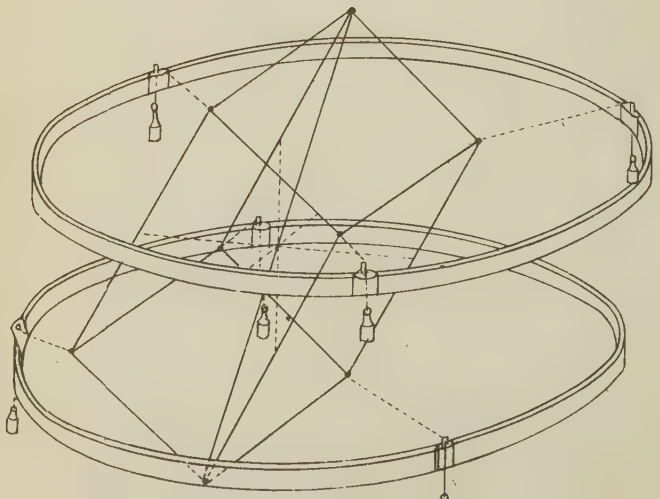

INTERMITTENT LIGHTNING FLASHES.

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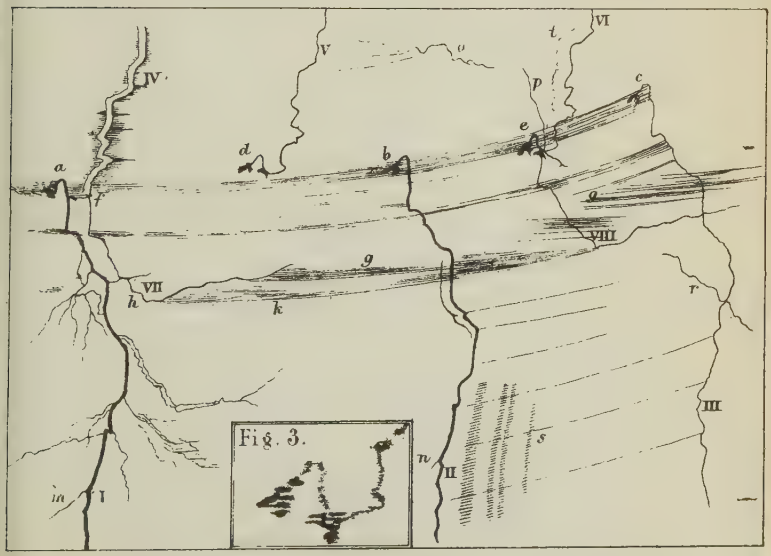

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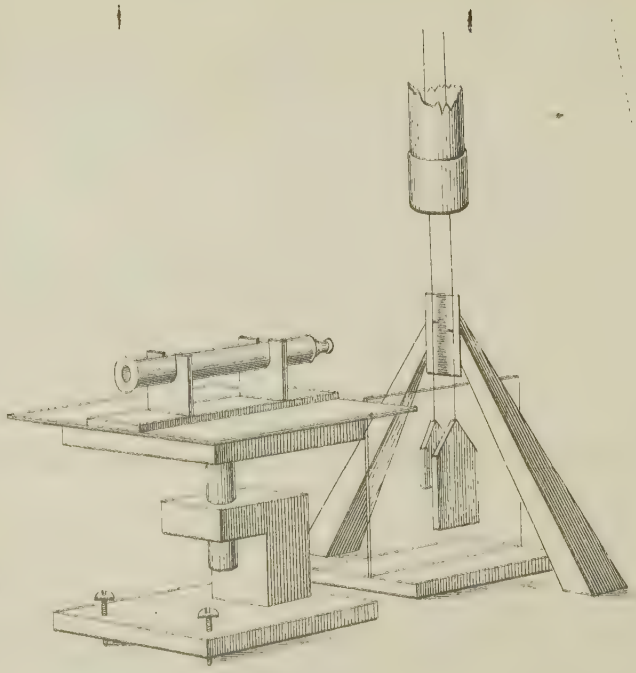

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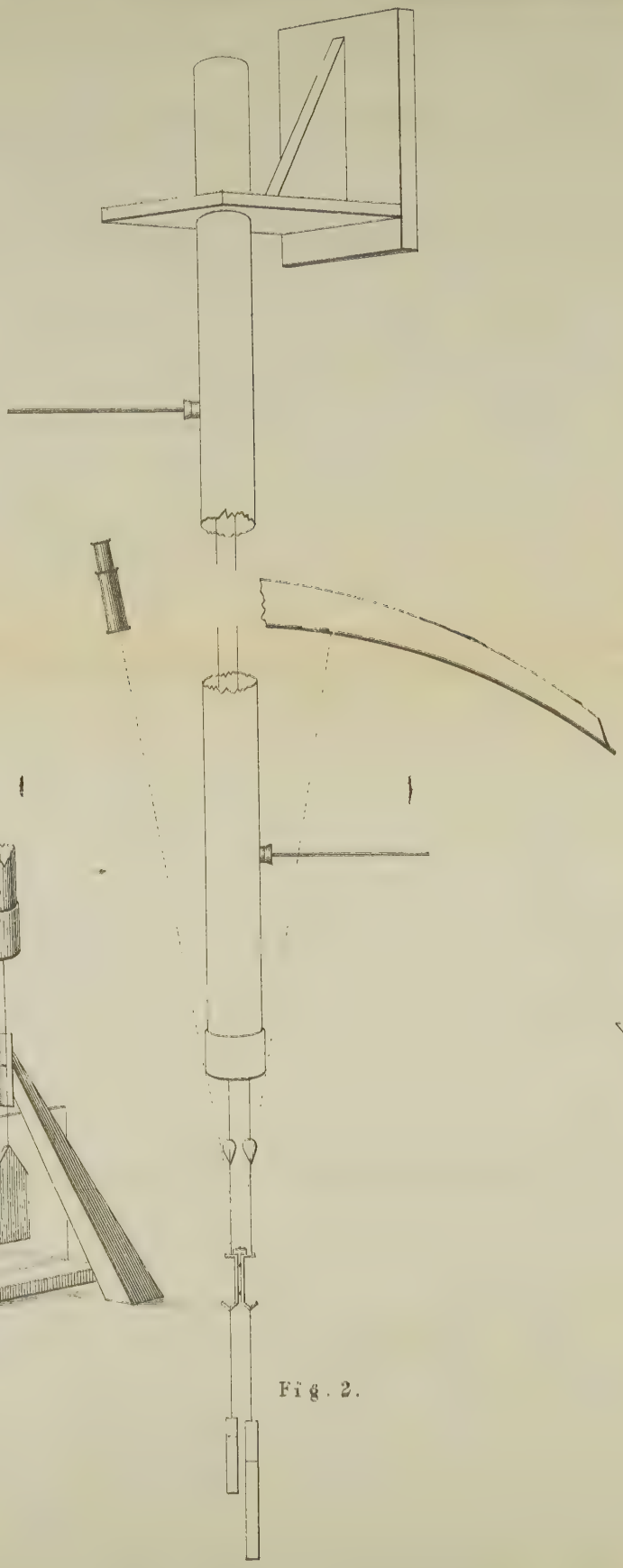


Fig. 2.

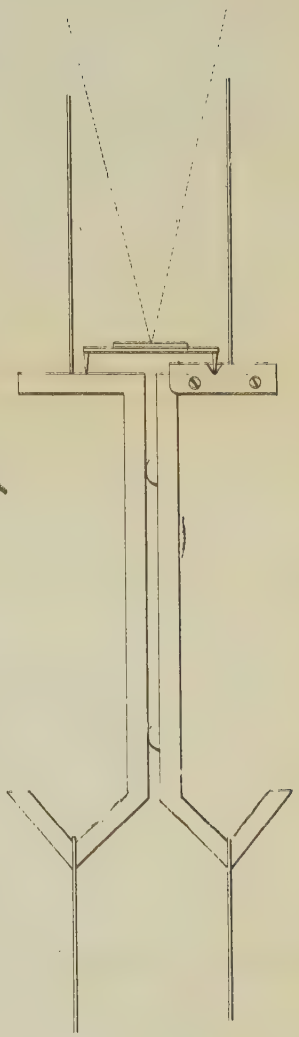
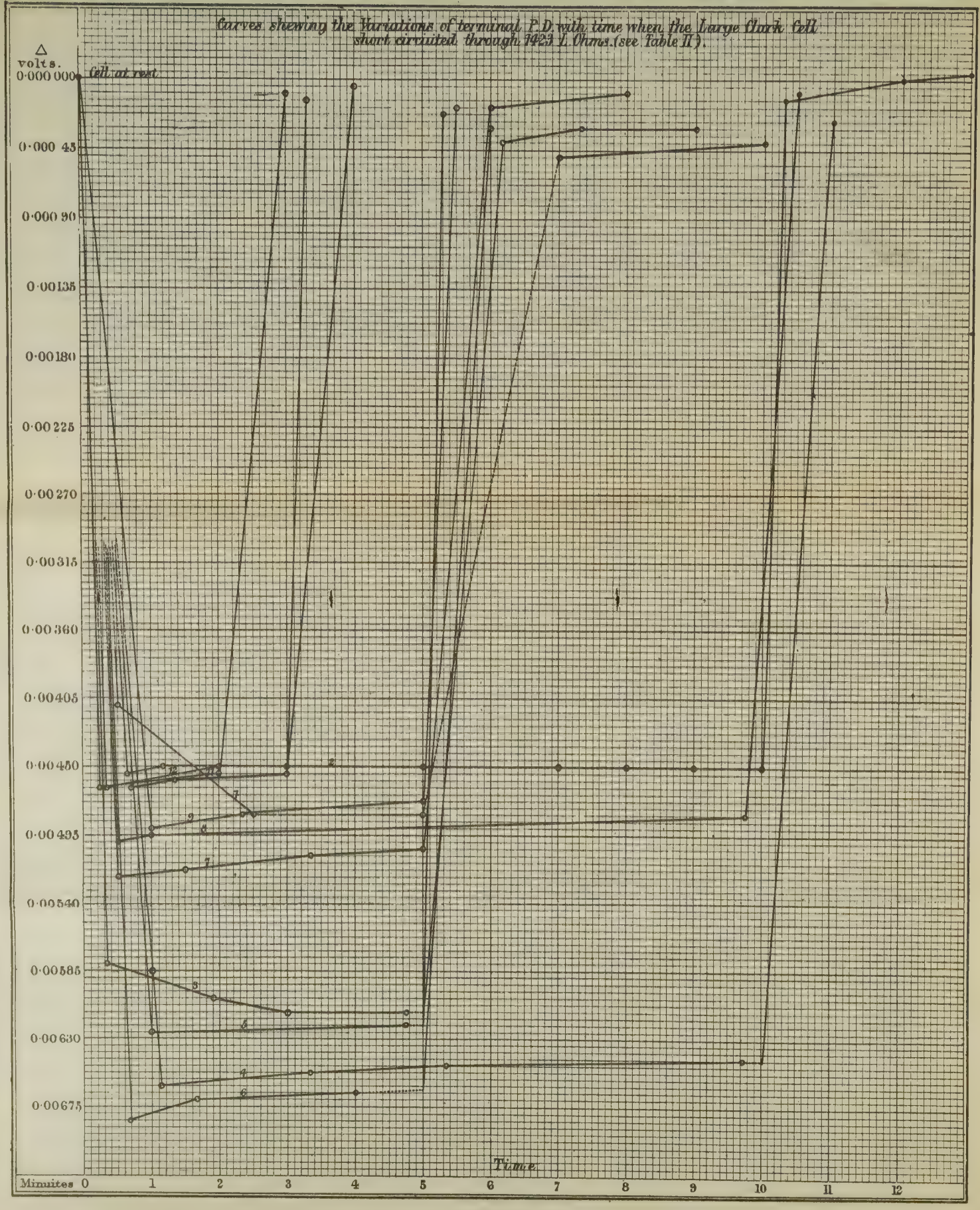


Fig. 3

Mintern Bros. lith.



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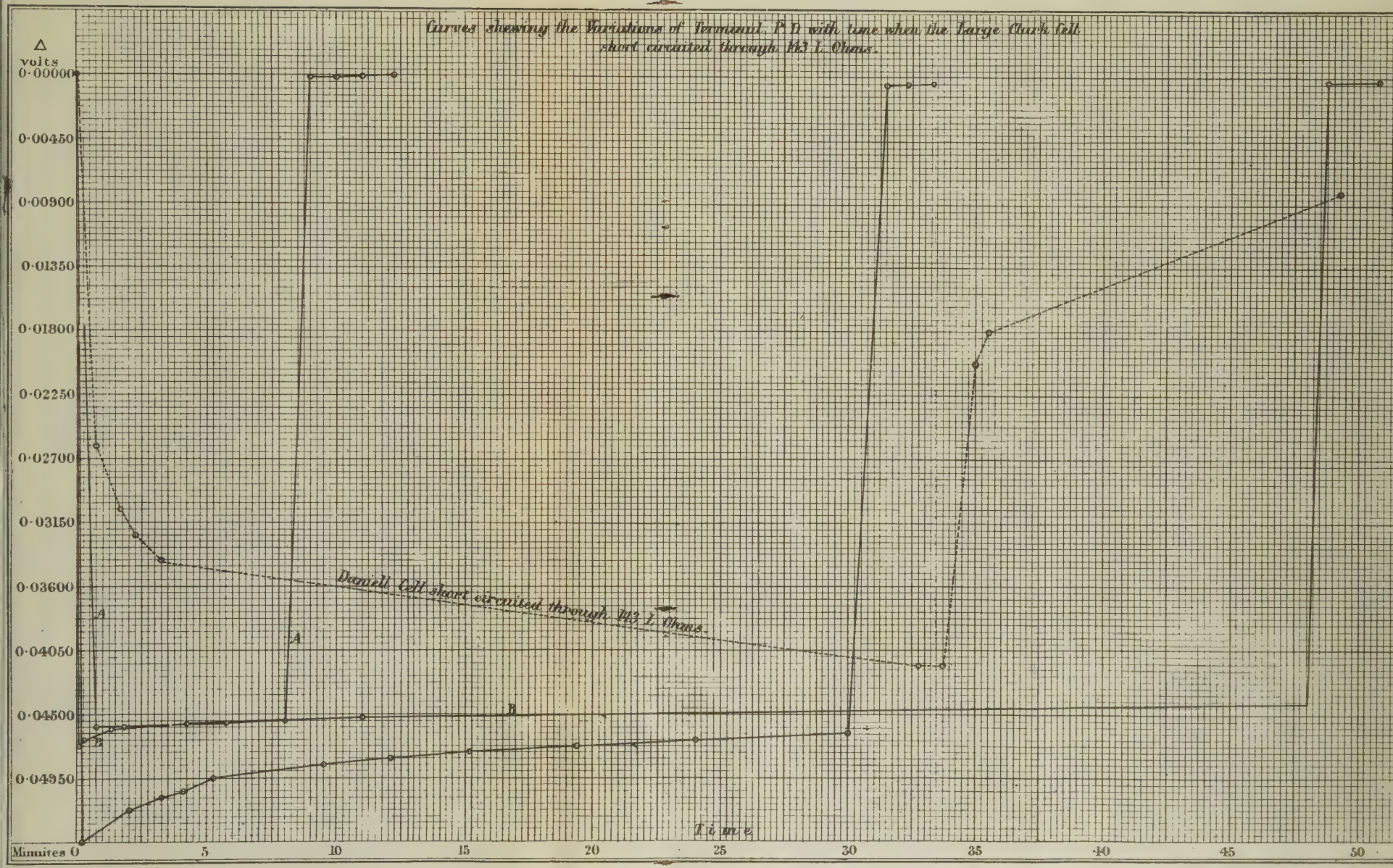
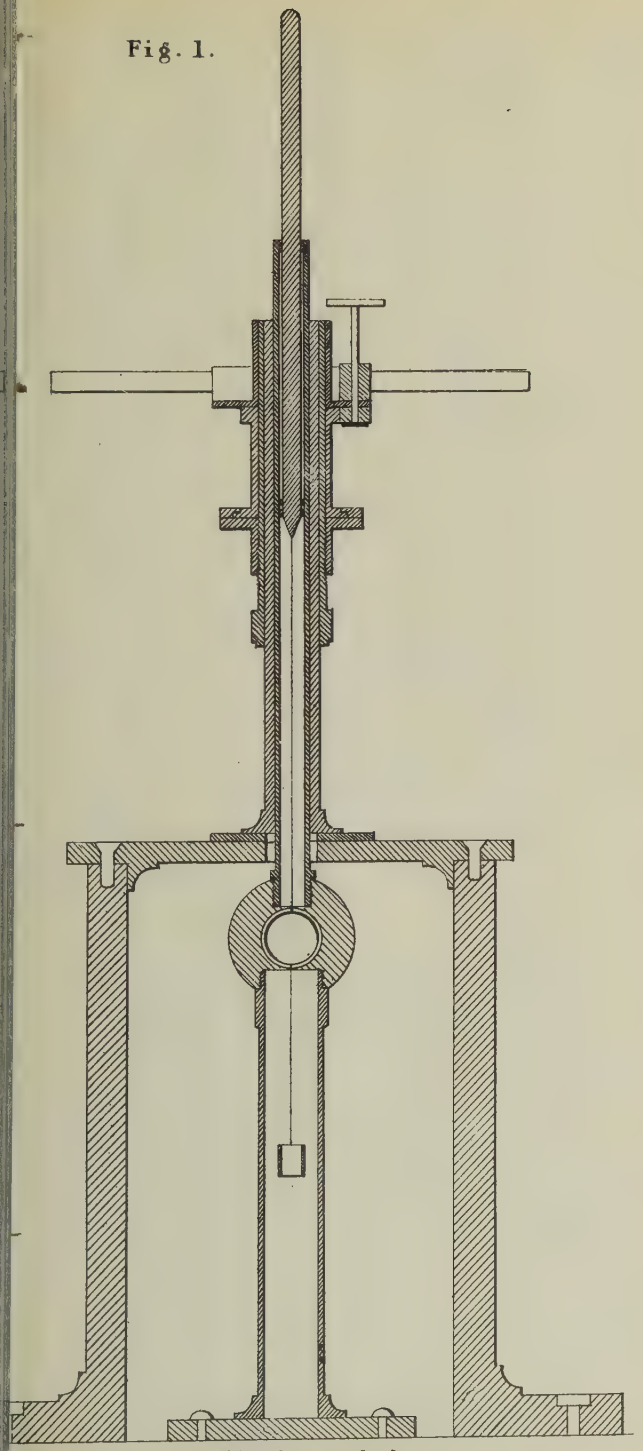


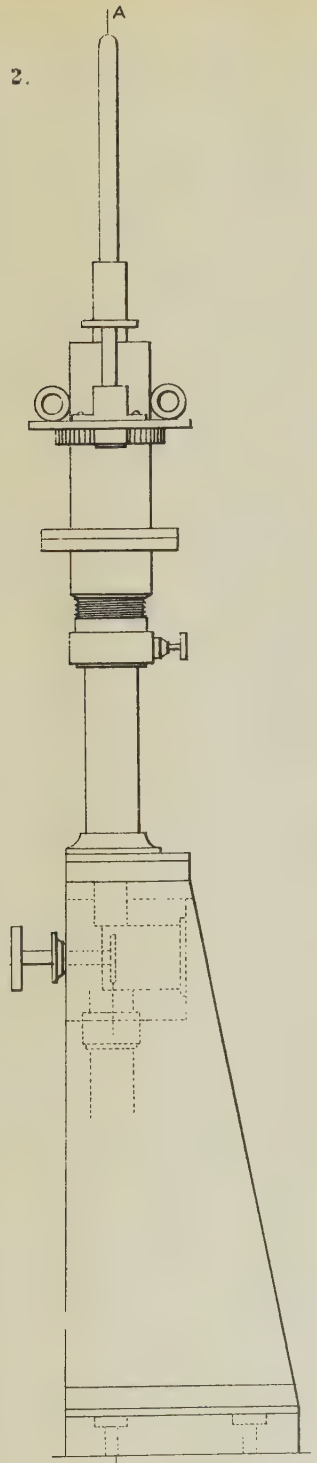
Mintern Bros lith.

Fig. 1.



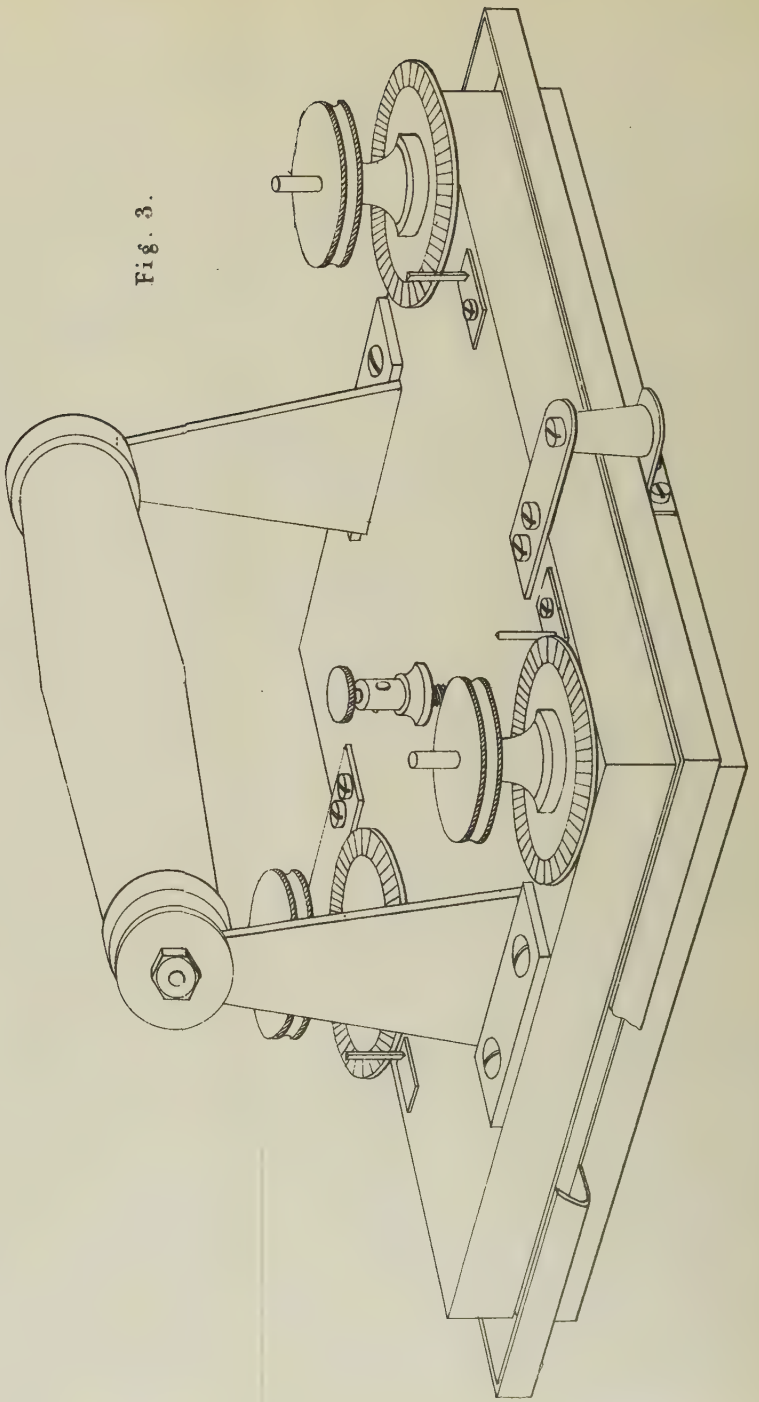
SECTION ON A. A.

Fig. 2.



A. SIDE ELEVATION.

Fig. 3.



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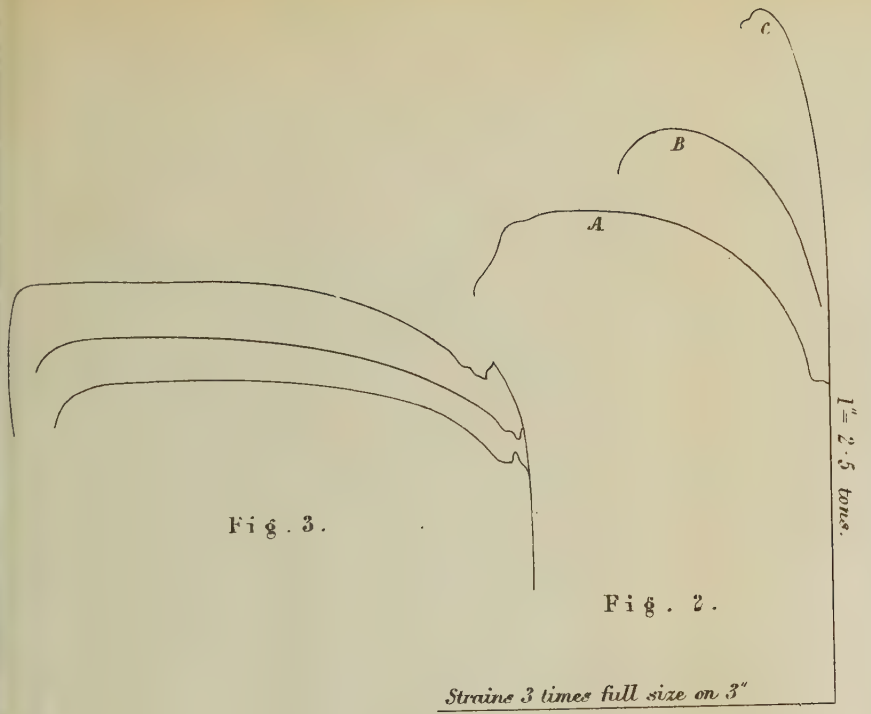

Fig. 3.

Fig. 2.

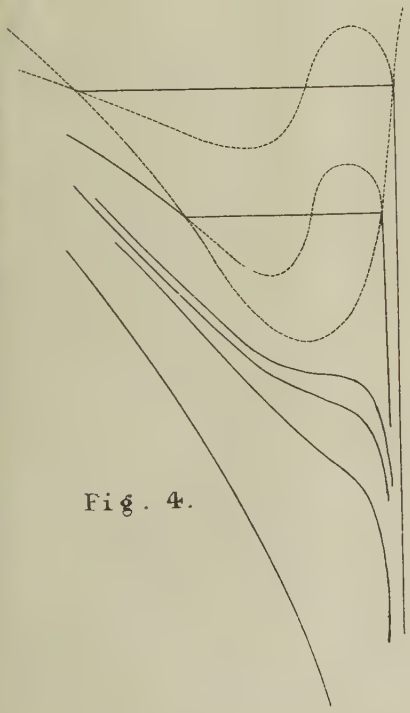

Fig. 4.

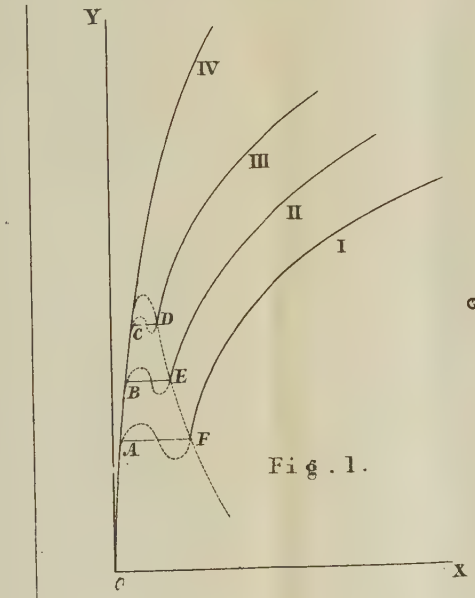

Fig. 1.

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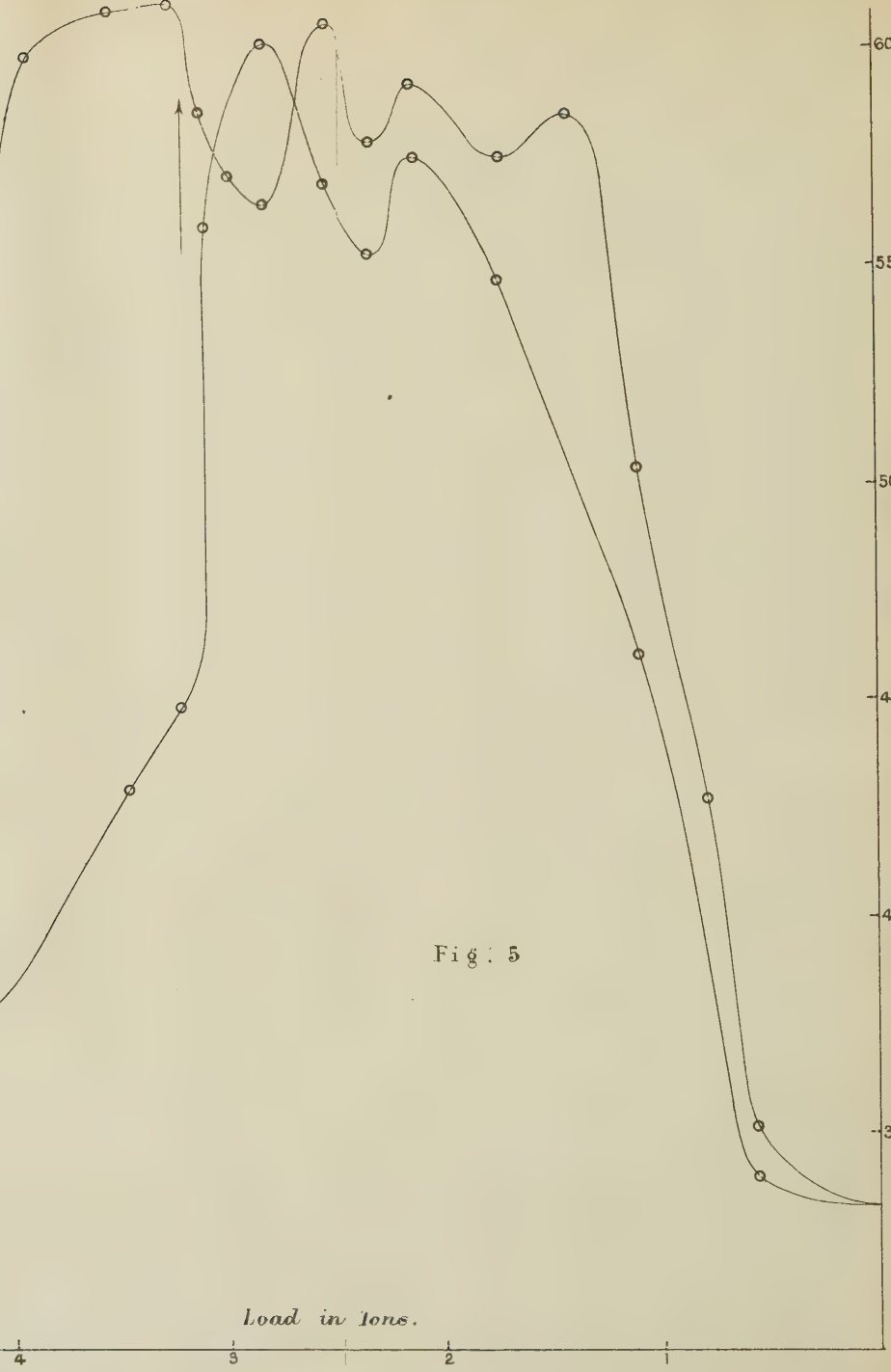


Fig. 5

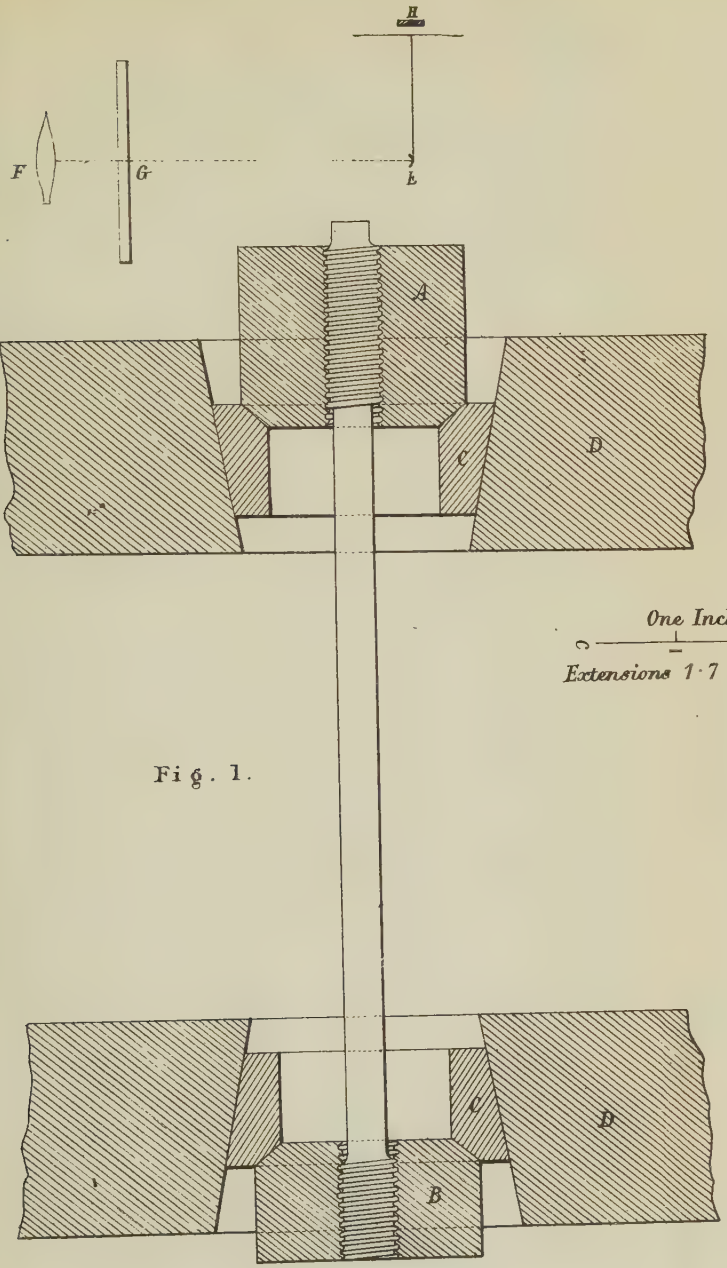
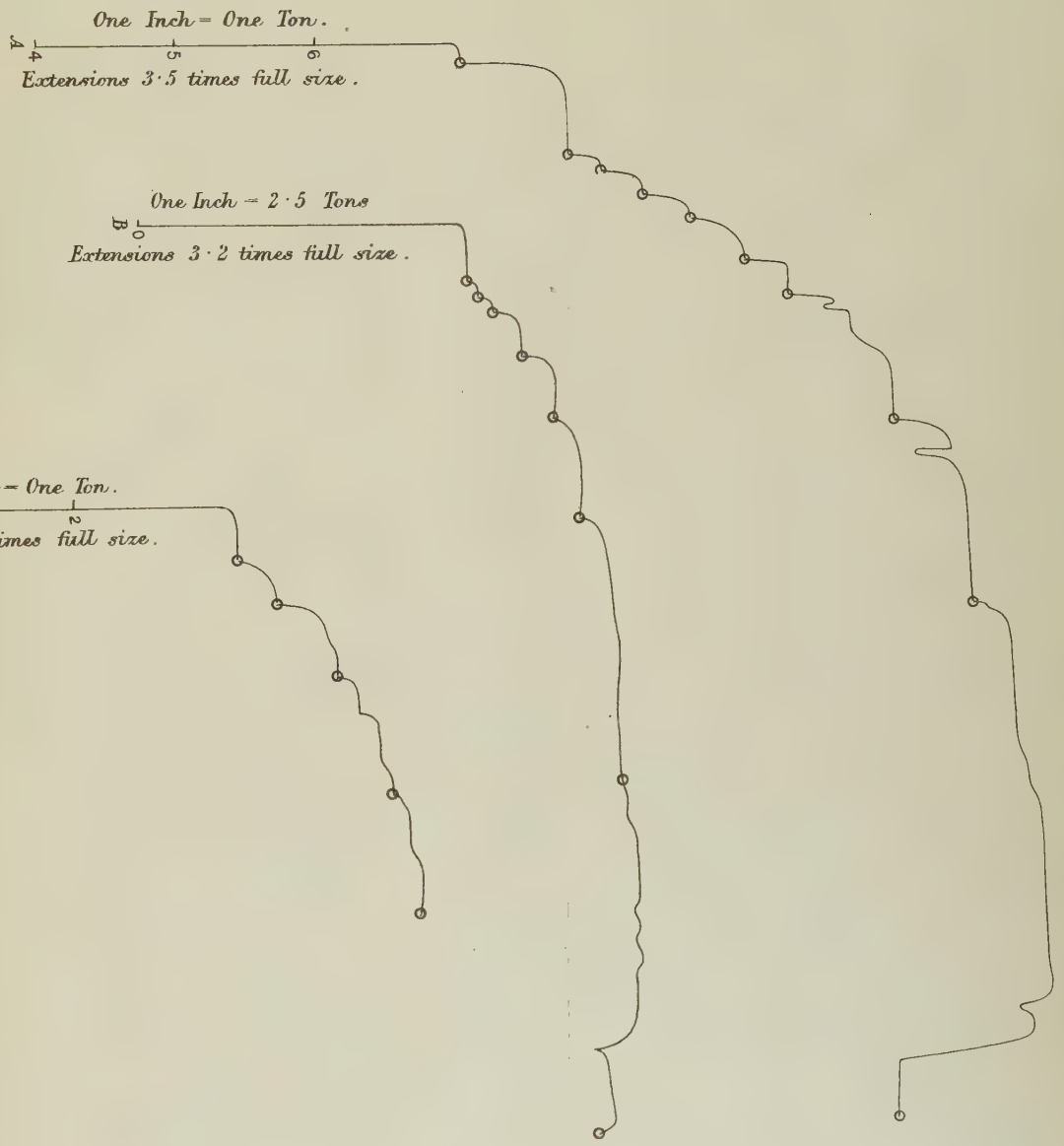


Fig. 1.



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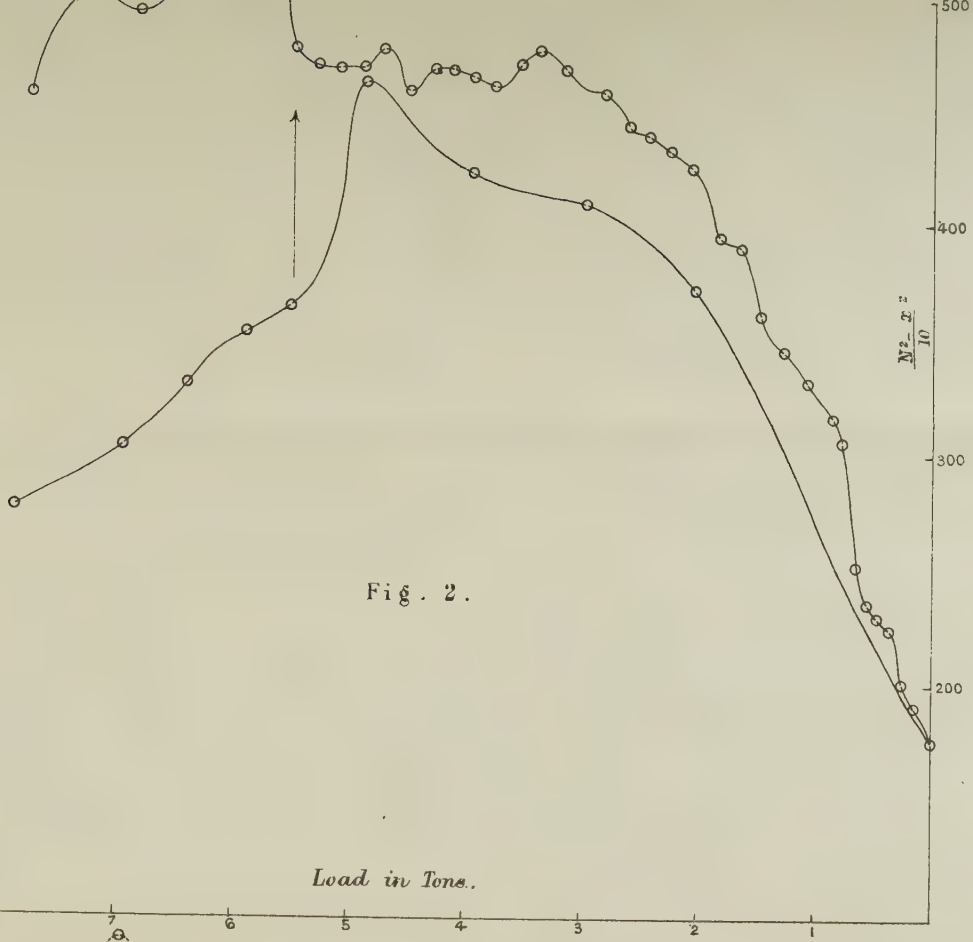


Fig. 2.

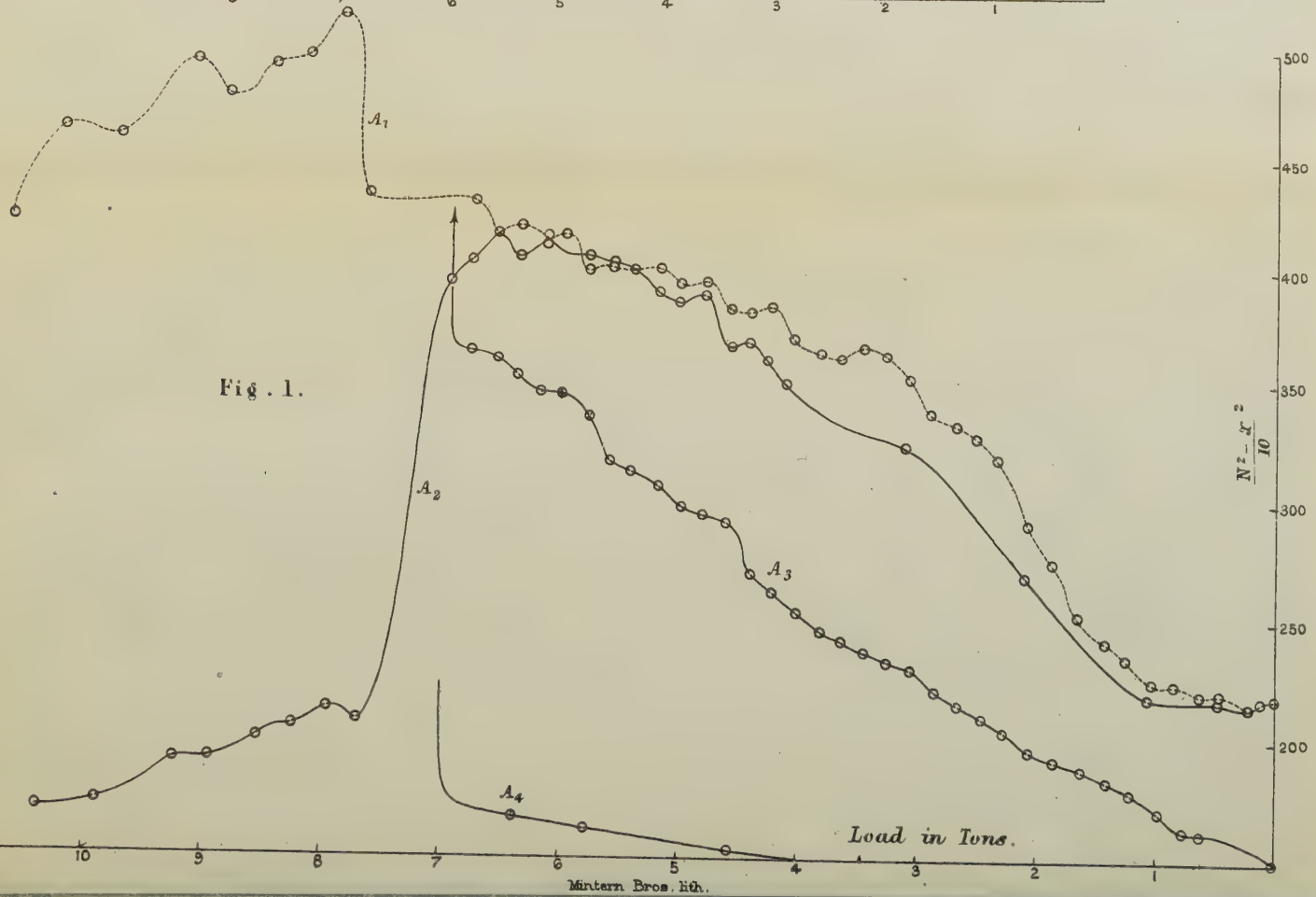


Fig. 1.

Mintern Bros. lith.

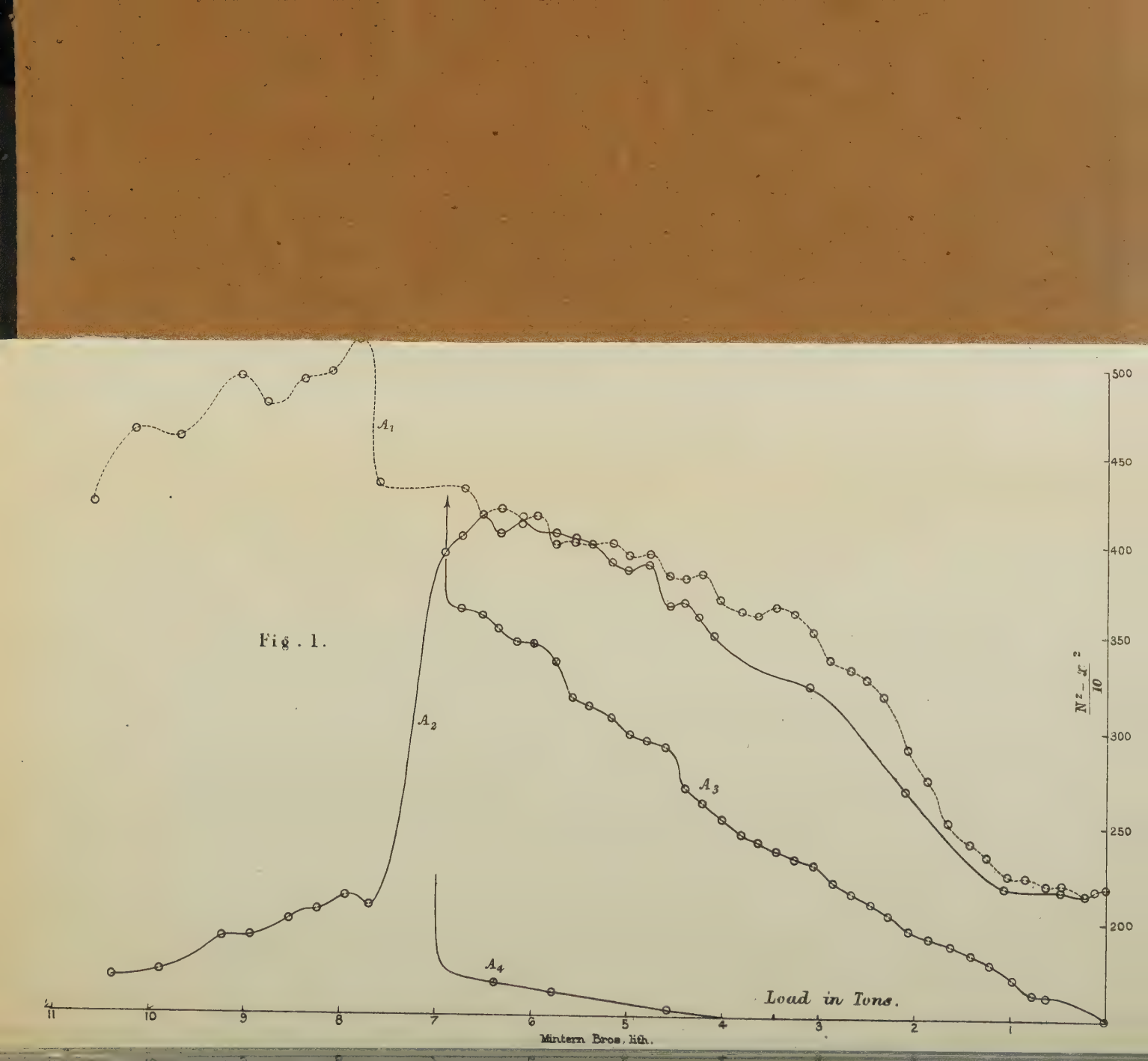

A1
A2
A3
A4
500
450
400
350
300
250
200
$\frac{N^2 - x^2}{10}$
11
10
9
8
7
6
5
4
3
2
1
Load in Tons.
Mintern Bros. lith.

Fig. 1.

Fig. 2.

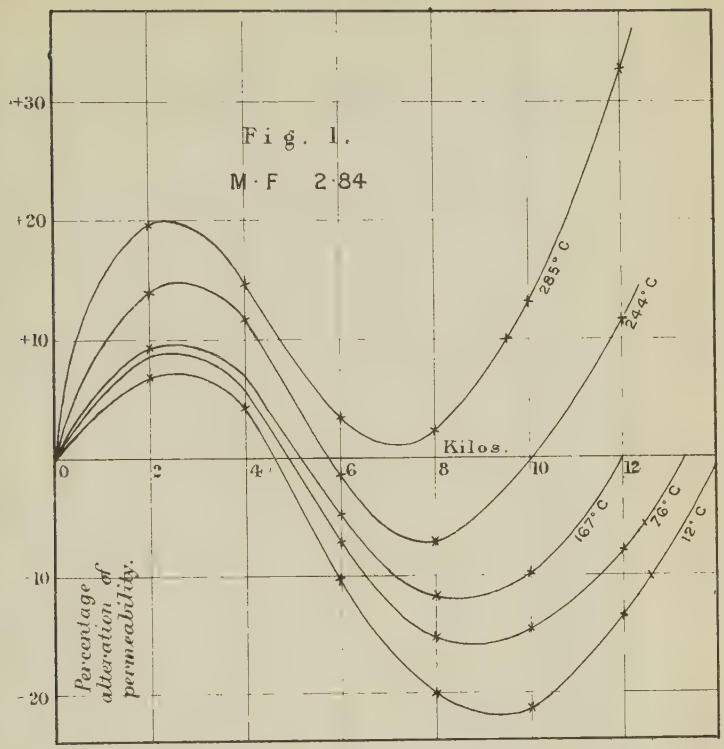


Fig. 1.
M·F 2·84
+30
+20
+10
-10
-20
0
2
4
6
8
10
12
Kilos.
Percentage alteration of permeability.
285° C
244° C
167° C
76° C
12° C

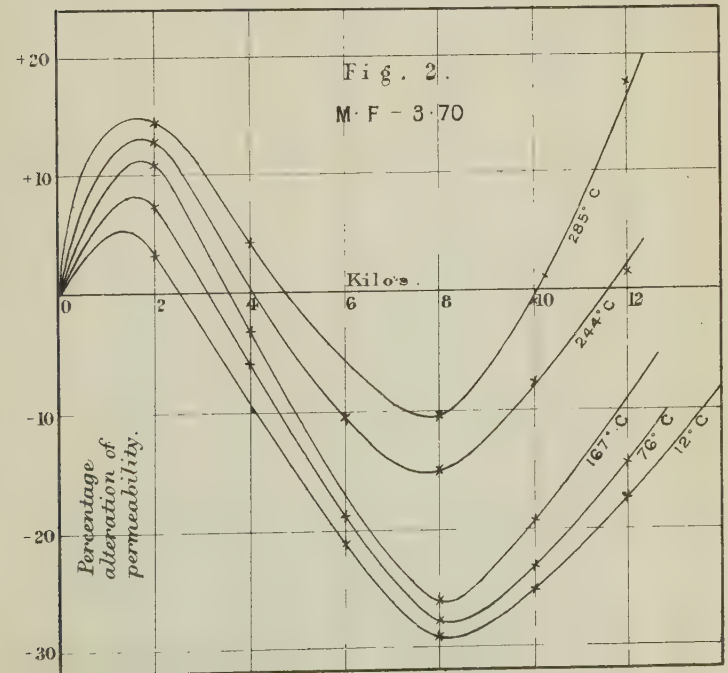


Fig. 2.
M·F = 3·70
+20
+10
-10
-20
-30
0
2
4
6
8
10
12
Kilos.
Percentage alteration of permeability.
285° C
244° C
167° C
76° C
12° C

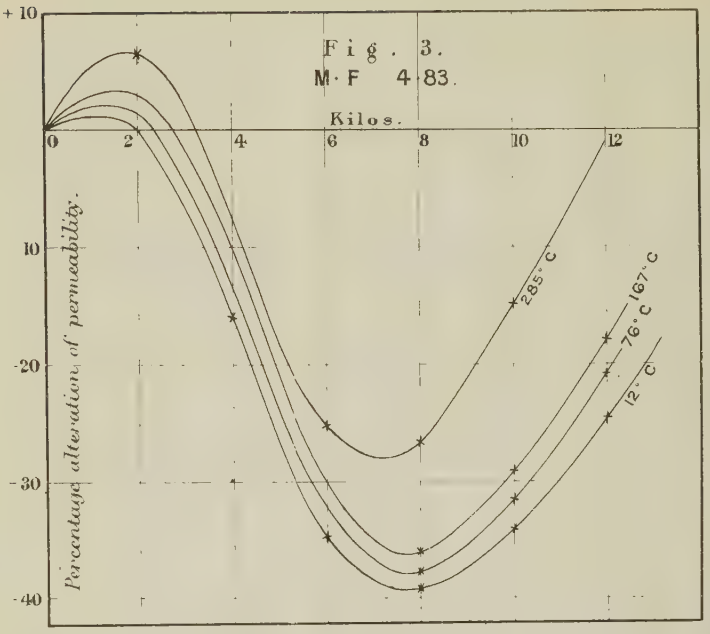


Fig. 3.
M·F 4·83.
+10
10
-20
-30
-40
0
2
4
6
8
10
12
Kilos.
Percentage alteration of permeability.
285° C
167° C
76° C
12° C

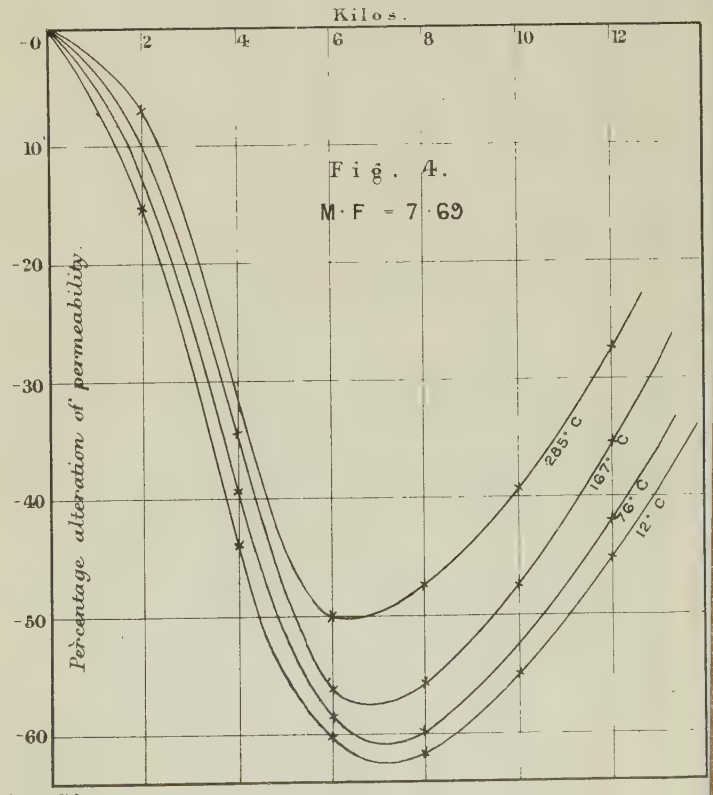
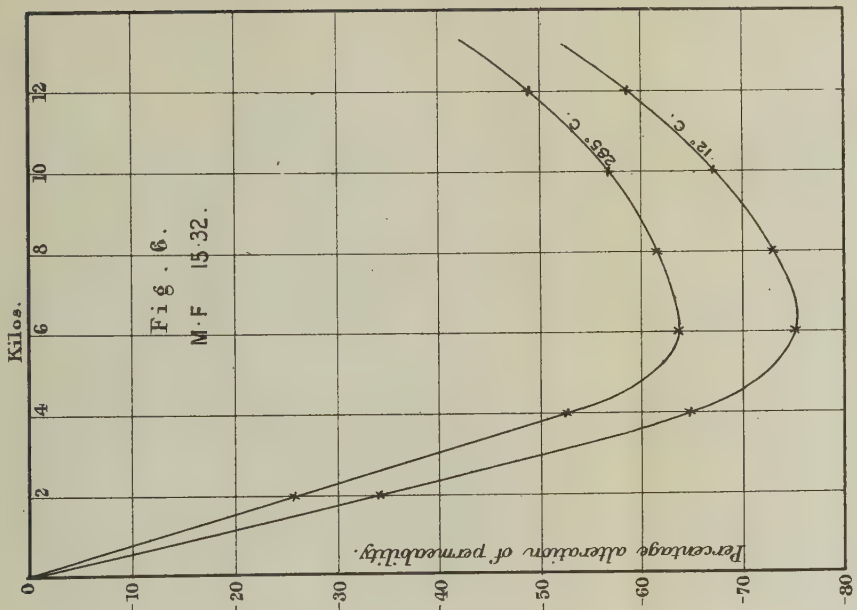
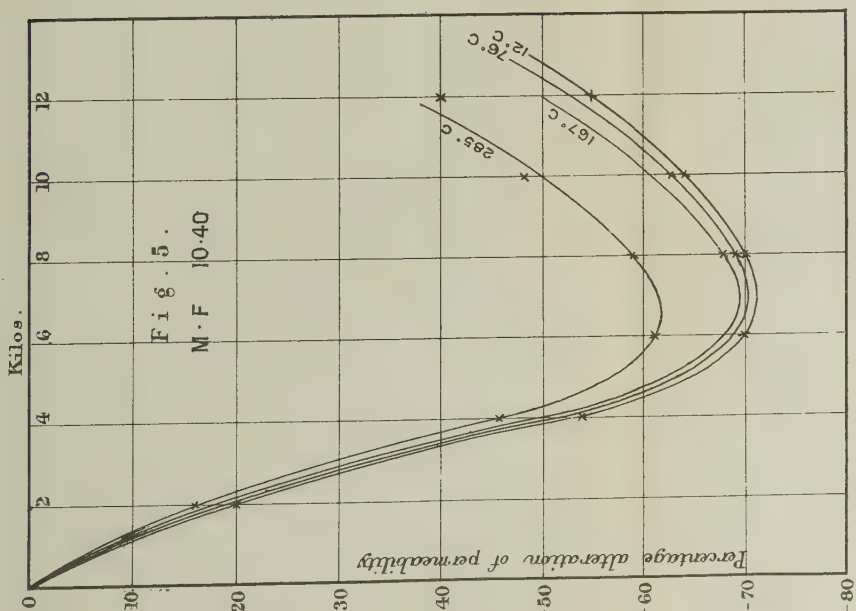


Fig. 4.
M·F = 7·69
-0
10
-20
-30
-40
-50
-60
0
2
4
6
8
10
12
Kilos.
Percentage alteration of permeability.
285° C
167° C
76° C
12° C





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PROCEEDINGS

AT THE

MEETINGS OF THE PHYSICAL SOCIETY

OF LONDON.

SESSION 1888-89.

February 11th, 1888.

Prof. A. W. Reinold, President, in the Chair.

The following communications were made:—

"On the Limit of Refraction in relation to Temperature and Chemical Composition." By the Rev. T. Pelham Dale.

"Note on the use of the term Resistance in the description of Physical Phenomena." By Mr. R. H. M. Bosanquet.

February 25th, 1888.

Prof. A. W. Reinold, President, in the Chair.

Mr. H. F. Newall was elected a Member of the Society.

The following communications were made:—

"Note on the Efficiency of Incandescent Lamps with Direct and Alternating Currents." By Profs. W. E. Ayrton and J. Perry.

"Observations on the Height, Length, and Velocity of Ocean Waves." By the Hon. Ralph Abercromby.

"On the Temperature at which Nickel begins suddenly to lose its Magnetic Properties." By Mr. HERBERT TOMLINSON.

"Experiments on Electrolysis." By Messrs. W. W. HALDANE GEE, H. HOLDEN, and C. H. LEES.

March 10th, 1888.

Prof. A. W. REINOLD, President, in the Chair.

Mr. CHARLES V. BURTON was elected a Member of the Society.

The following communications were made:—

"On a Compact Form of Reflecting-Galvanometer, Lamp, and Scale." By Mr. G. L. ADDENBROOKE.

"On the Measurement of the Power supplied to the Primary Coil of a Transformer." By Mr. E. C. RIMINGTON.

"On the Magnetic Circuit of Dynamo Machines." By Profs. W. E. AYRTON and J. PERRY.

"Note on the Employment of an Electro-dynamometer for determining the Difference of Phase of two Harmonic Currents of Electricity." By Mr. T. H. BLAKESLEY.

April 14th, 1888.

Mr. SHELFORD BIDWELL, Vice-President, in the Chair.

The following communications were made:—

"On the Variation of the Coefficients of Induction." By Mr. W. E. SUMPNER.

"Experiments on Soap Bubbles." By Mr. C. V. BOYS.

April 28th, 1888.

Prof. A. W. REINOLD, President, in the Chair.

The following were elected Members of the Society:—

Mr. A. W. SLATER and Mr. W. H. SNELL.

The following communications were made:—

"On Electromotive Force by Contact." By Mr. C. V. BURTON.

"On a Theory concerning the sudden loss of Magnetic Properties in Iron and Nickel." By Mr. HERBERT TOMLINSON.

"Note on the Graphic Treatment of the Lamont-Frölich for Induced Magnetism." By Prof. S. P. THOMPSON.

May 12th, 1888.

Prof. A. W. Reinold, President, in the Chair.

The following were elected Members of the Society :—

Mr. Henry H. Cunynghame and Mr. G. L. Addenbrooke.

The following communications were made :—

"Note on the Conditions of Self-Excitation in a Dynamo-Machine." By Prof. S. P. Thompson.

"Note on the Conditions of Self-Regulation in a Constant Potential Dynamo-Machine." By Prof. S. P. Thompson.

"On Magnetic Lag, and the Work lost due to Magnetic Lag in Alternating-Current Transformers." By Mr. T. H. Blakesley.

"On a Simple Apparatus for the Measurement of the Coefficients of Expansion by Heat." By Profs. W. E. Ayrton and J. Perry.

May 26th, 1888.

Mr. Shelford Bidwell, Vice-President, in the Chair.

Mr. S. O. Roberts was elected a Member of the Society.

The following communications were made :—

"Note on the Governing of Electromotors." By Profs. W. E. Ayrton and J. Perry.

"On the Formulæ of Bernouilli and Haecker for the Lifting-Power of Magnets." By Prof. S. P. Thompson.

"Experiments on Electrolysis.—Part II. Irreciprocal Conduction." By Messrs. W. W. Haldane Gee and H. Holden.

June 9th, 1888.

Prof. A. W. Reinold, President, in the Chair.

Mr. James Livesey Howard was elected a Member of the Society.

The following communications were read :—

"On the Analogy between Dilute Solutions and Gases as regards Gay-Lussac's and Boyle's and Avogadro's Laws." By Prof. van't Hoff. Presented by Prof. W. Ramsay.

"On a Method of comparing very unequal Capacities." By Dr. A. H. Fison.

"On a new form of Lantern." By Mr. W. Lant Carpenter.

"Note on some Additions to the Kew Magnetometer." By Profs. T. E. Thorpe and A. W. Rücker.

June 23rd, 1888.

Prof. A. W. Reinold, President, in the Chair.

The following communications were made :—

"On the Photometry of Colours." By Capt. W. de W. Abney.

"Note on Continuous Current Transformers." By Prof. S. P. Thompson.

"On an Optical Model." By Prof. A. W. Rücker.

"On a new Barometer." By Mr. T. H. Blakesley.

"On the Existence of an Undulatory Movement accompanying the Electric Spark." By Dr. Ernest H. Cook.

November 10th, 1888.

Prof. A. W. Reinold, President, in the Chair.

The following communications were made :—

"On the Calculation of the Coefficient of Mutual Induction of a Circle and a Coaxal Helix." By Prof. J. V. Jones.

"On the Upper Limit of Refraction in Selenium and Bromine." By the Rev. T. Pelham Dale.

"Experiments on Glass in Polarized Light." By Prof. S. P. Thompson.

"On a New Form of Standard Resistance-Coil." By Dr. J. A. Fleming.

November 24th, 1888.

Prof. A. W. Reinold, President, in the Chair.

The following were elected Members of the Society :—

Rev. Thomas Pelham Dale and Dr. Robert M. Walmsley.

The following communications were made :—

"On the Measurement of the Luminosity of Coloured Surfaces." By Capt. W. de W. Abney.

"On the Suppressed Dimensions of Physical Quantities." By Prof. A. W. Rücker.

December 8th, 1888.

Prof. A. W. Reinold, President, in the Chair.

The following were elected Members of the Society :—

Mr. A. P. Trotter and Dr. A. Gamgee, F.R.S.

The following communications were made :—

"Note on a Modification of the ordinary Methods of Determining Electro-Magnetic Capacity." By Dr. J. W. W. Waghorn.

"On some Facts connected with the Systems of Scientific Units of Measurement." By Mr. T. H. Blakesley.

"On some Improved Polarizing Apparatus for Microscopes." By Prof. S. P. Thompson.

January 26th, 1889.

Prof. F. Fuller, Member of Council, in the Chair.

The following communications were made :—

"Notes on Polarized Light: viz. On the Structure of Natural Diffraction-Gratings of Quartz; on Ahren's Modification of Delezenne's Polarizer; and on the Use of two Quarter-wave Plates in Combination with a Stationary Polarizer." By Prof. S. P. Thompson.

"Note on a Relation between Magnetization and Speed in a Dynamo-Machine." By Prof. S. P. Thompson.

"On the Divergence of Electromotive Forces from Thermochemical Data." By Prof. E. F. Herroun.

Annual General Meeting.

February 9th, 1889.

Prof. A. W. Reinold, President, in the Chair.

The following Report of the Council was read by the Chairman :—

The Council, in presenting their Annual Report, congratulates the Society on the success which their Meetings have maintained during the past year. The attendance at the Meetings has been from 25 to 40 Members and Visitors, and an ample supply of interesting papers has been brought forward. The Meetings have been somewhat shorter than in former years, as the Council have found that

it was inconvenient to Members to have the Meetings prolonged much beyond half-past 4 o'clock.

The number of Members of this Society has increased to 434; but the Council, believing that if the advantages offered by the Society were more generally known a much greater increase might be reasonably expected, have had a fly-leaf in the following terms printed and circulated to the Members:—

"THE PHYSICAL SOCIETY OF LONDON.

"This Society was founded in 1874 for the purpose of receiving and discussing communications relating to Physics, and for the exhibition of apparatus for Physical research, and experiments illustrative of Physical phenomena, and for the publication of communications made to the Society, and other papers relating to Physics.

"The Society numbers between 300 and 400 Members.

"The Meetings are held on the 2nd and 4th Saturdays in each month from November to June inclusive, except that two meetings are omitted at Christmas and one at Easter.

"The time of meeting is 3 P.M.

"The place of meeting is the Physical Lecture-room at the Science Schools, South Kensington. This room is admirably suited for the exhibition of experiments, and the use of apparatus in the large collection belonging to the Science Schools is accorded to those who desire to exhibit experiments at the Society's Meetings; assistance is also rendered by the Demonstrator and his Staff in the preparation of such experiments.

"Every paper communicated to the Society is now published either in full or in abstract. If published in full it is published in the first instance in the 'Philosophical Magazine,' as communicated by this Society, and 100 copies of the paper are sent to the author. Papers which have thus appeared in the 'Philosophical Magazine' are afterwards reprinted in the Proceedings of this Society. Papers which the Council desire not to publish in full are published in abstract in the Proceedings of the Society.

"The Proceedings are as a rule published quarterly, and are issued gratis to Members.

"Besides the Proceedings, the Society from time to time issues other publications which are also supplied gratis to Members at the time of issue, and which may afterwards be purchased by Members at lower terms than by the public.

"The works of Wheatstone and of Joule have been thus published by the Society, and a series of translations of foreign papers has been commenced.

"The Library belonging to the Society is kept in a room adjoining the place of meeting. Application for leave to take out books should be made to the Librarian.

"The Entrance Fee is £1, and the Annual Subscription is also £1. Annual Subscriptions may be compounded for by the payment of £10 in one sum.

"Letters for the Officers of the Physical Society can be addressed to the Physical Laboratory, South Kensington, S.W.

"Members having friends outside the Society who are interested in the study of Physics are invited to draw their attention to the advantages which the Society offers.

"Additional copies of this fly-leaf can be obtained by application to the Secretaries."

In the past year the Society has issued to its Members the first part of its new undertaking, the Translation of important Physical Memoirs. This part contains five important papers by Prof. von Helmholtz on the Application of the Principles of Thermodynamics to Chemical Action and Electrolysis.

It was hoped that the translation of Puluj's memoir on Radiant Electrode-Matter would have also been issued; but for a more complete treatment of this subject it has been found desirable to include two additional papers by Hittorf on the Conduction of Electricity in Gases. The translation of them is well forward, and the whole part, it may be expected, will be in the hands of the Members early in the present session.

Prof. Threlfall, of the University of Sydney, having offered to the Society the translation by himself and Mr. Adare of Van der Waals' important memoir on the Continuity of the Gaseous and Liquid States of Matter, the Council has accepted the proposal with thanks, and has decided to undertake the publication of this translation.

A Committee, with the valuable help of Mr. H. H. Cunynghame, has settled what portions of Volta's work are to be undertaken, and the translation is being proceeded with. It must, however, be stated that the production of the above memoir will quite absorb whatever of the funds of the Society are available for this purpose in the present year.

The Council have observed with satisfaction the admirable manner

in which the Meetings of the Society have been reported in the weekly periodicals, and they have directed that the reports of the General Meetings, as furnished by the Society's reporter, shall be published in the 'Proceedings,' in so far as the discussions are concerned, and that when a paper for any reason is not published *in extenso*, the abstract furnished by the reporter shall be published in the 'Proceedings.'

The following additions have been made to the Library of the Society:—

Newspapers:—

Nature.
The Electrical Review.
Engineering.
The Electrician.
The Electrical Engineer.
Scientific News.
Electrical Plant.
Ingeniero y Ferrotero Español.
Crónica Cientifica.

Magazines.

The Philosophical Magazine.
Beiblätter der Physik und Chemie.
Annalen der Physik und Chemie.
Journal de Physique.

Journals &c. of Societies:—

Royal Society, Proceedings.
Society of Arts, Journal.
Society of Telegraph Engineers, Journal.
Mechanical Engineers, Proceedings.
Royal Meteorological Society, Quarterly Journal.
Cambridge Philosophical Society, Proceedings.
Middlesex Natural History Society, Transactions.
Philosophical Society of Glasgow, Proceedings.
Manchester Philosophical and Literary Society, Proceedings.
Royal Dublin Society, Proceedings and Transactions.
Royal Society of New South Wales, Journal.
Canadian Institute, Proceedings.
Academy of Natural Sciences, Philadelphia, Proceedings.
Physical and Chemical Society of Russia, Journal.

Physical Society of Berlin, Proceedings.
Société Française de Physique, Séances.
College of Science of Japan, Journal.
Seismological Society of Japan, Transactions.
Atti R. Accad. Sci. Torino.

Books and Papers received :—

University College, London, Calendar.
Glasgow University, Calendar.
Imperial University of Japan, Calendar.
Geological Society, Report.
Observations, Hongkong.
Magnetic and Meteorological Observations, Bombay.
Institute of Civil Engineers, List of Members.
A Practicable Decimal System.
Four papers on Differential Equations, by Asutosh Mukhopadhyay.
University Studies, Nebraska.
Catalogue of Additions to Radcliffe Library, Oxford.
Life of Sir William Siemens.
Norwegian North-Sea Expeditions.
Hadfield on Manganese Steel.
Catalogue of Canadian Plants (Endogens).
List of Publications of Geological and Natural History Survey of Canada.
Prace Matematyezno-Fizyezne. Vol. I.
Stonyhúrst College Observations.
Yorkshire College, Leeds, 14th Report.
Kew Observatory Report.
The Planets and The Cause of Light (Carruthers).
Sachregister Annalen, 1824–1877.
La Puissance Motrice du Feu (Carnot), given by Rev. A. Freeman.
Separatabdruck, by Dr. H. Bunte.
La Théorie Cinétique des Phénomènes Capillaires.

The set of the 'Philosophical Magazine' in the Society's Library has been completed by the purchase of the volumes for the four missing years, Messrs. Taylor and Francis having sold them to the Society at the lowest trade price, and having promised to present future numbers free of cost.

The Council have also received promises from the publishers of

'Nature,' the 'Electrician,' the 'Electrical Engineer,' and 'Engineering,' to send their future numbers free of cost.

So far as the Council are aware no death has occurred among the Ordinary Members of the Society but the Council regret to have to announce the death of a distinguished Honorary Member, Prof. Clausius; and they nominate in his place Prof. Robert Wilhelm Bunsen, of Heidelberg, whose name is a household word in every Physical and Chemical Laboratory.

Rudolf Julius Emanuel Clausius was born January 2, 1822, at Cöslin, in Pomerania. About the year 1840 he entered the University of Berlin, in which he afterwards became a "Privat-Docent." After holding for some time the post of Instructor in Physics in the Artillery School at Berlin, he went in 1855 to Zurich, where he was Professor of Physics first in the Polytechnic and afterwards in the University. In 1867 he migrated to Wurtzburg; and in 1869 he was appointed Professor of Physics in the University of Bonn. This post he retained till his death in 1888.

His first paper, in Poggendorff's 'Annalen,' was entitled "Ueber die Lichtzerstreuung in der Atmosphare und über die Intensität des durch die Atmosphäre reflectirten Sonnenlichts" (Pogg. *Ann.* vol. lxxii. p. 294), and he returned to the same or kindred subjects several times afterwards.

He will, however, be chiefly famous as one of the founders of the modern theory of Thermodynamics, in the establishment of which he played a part of the first importance.

It would be scarcely possible to give a detailed account of his work without entering on a somewhat extended history of the development of the Theory of Heat in the middle of the present century, and on a discussion as to questions of priority which have been somewhat hotly debated, and which would hardly be in place in a brief obituary notice. It is, however, pleasant to remember that in the last edition of his 'Theory of Heat,' published in 1887, Clausius expressed himself as fully satisfied with the treatment he had received at the hands of Sir W. Thomson and Maxwell, our countrymen who have done most to develop the subject in the same direction as that in which he himself worked.

In addition to his work on the 'Second Law of Thermodynamics' and on the 'Theory of Gas,' Clausius introduced the use of the Virial, a function of great generality, which has found an important application in the establishment of relations between the mean energy of molecular motions and intermolecular forces. In

his work on the 'Anwendung der mechanischen Wärmetheorie zu Grunde liegenden Principien auf die Electricität,' he established an interesting equation, by which the ratio of the volume of the molecules to the total apparent volume of a dielectric is calculated from a knowledge of its inductive capacity. The results are in many cases in close accord with those deduced from the corresponding formula obtained from optical considerations by H. A. and L. Lorentz, and in the case of the gases are of the same order as the values given by the theory of Van der Waals.

Apart from his scientific work, the life of Clausius appears to have been devoid of stirring incident. Though his merits were recognized early, promotion—as the above record shows—did not come very quickly.

His 'Theory of Heat' has twice been translated into English, first by Dr. Hirst and afterwards by Mr. W. R. Browne. He was elected an Honorary Member of the Physical Society on February 14, 1880.

The adoption of the Report was proposed from the Chair, and carried unanimously.

The Society then proceeded to ballot for the Honorary Member nominated by the Council, and Professor Robert Wilhelm Bunsen was unanimously elected an Honorary Member of this Society.

The election of Officers and other Members of Council then took place, and the new Council was constituted as follows:—

President.—Prof. A. W. Reinold, M.A., F.R.S.

Vice-Presidents who have filled the office of President.—Dr. J. H. Gladstone, F.R.S.; Prof. G. C. Foster, F.R.S.; Prof. W. G. Adams, M.A., F.R.S.; Sir William Thomson, D.C.L., LL.D., F.R.S.; Prof. R. B. Clifton, M.A., F.R.S.

Vice-Presidents.—Dr. E. Atkinson; Prof. W. E. Ayrton, F.R.S.; Shelford Bidwell, M.A., LL.B., F.R.S.; The Right Hon. Lord Rayleigh, M.A., D.C.L., Sec.R.S.

Secretaries.—Walter Baily, M.A.; Prof. J. Perry, D.Sc., F.R.S.

Treasurer.—Prof. A. W. Rücker, M.A., F.R.S.

Demonstrator and Librarian.—C. V. Boys, F.R.S.

Other Members of Council.—Hon. R. Abercromby; T. H. Blakesley, M.A., M.I.C.E.; W. H. Coffin; Conrad W. Cooke; Prof. O. Lodge, D.Sc., F.R.S.; Prof. W. Ramsay, Ph.D., F.R.S.; W. N. Shaw, M.A.; Prof. S. P. Thompson, D.Sc.; H. Tomlinson, B.A.; G. M. Whipple, B.Sc.

Votes of thanks were passed to the Lords Committee of the Council on Education; to the Officers; and to the Auditors.

The Treasurer in Account with the Physical Society, from January 1st, 1888, to December 31st, 1888.

Dr.	£	*s.*	*d.*	£	*s.*	*d.*
Balance in Bank, January 1, 1888				128	19	3
Entrance-Fees	10	0	0			
Subscription for 1881	1	0	0			
Subscriptions for 1882	1	0	0			
„ 1883	1	0	0			
„ 1884	1	0	0			
„ 1885	1	0	0			
„ 1886	2	0	0			
„ 1887	5	0	0			
„ 1888	117	0	0			
„ 1889	4	0	0			
Excess Subscriptions	0	3	0			
Six Life Subscriptions	60	0	0			
				203	3	0
One year's Dividend on £400 Furness 4 per cent. Debenture Stock, less Income Tax	15	11	0			
One year's Dividend on £575 Midland 4 per cent. Preference Stock, less Income Tax	22	7	1			
One year's Dividend on £200 Lancaster Corporation Stock, less Income Tax	7	15	6			
One year's Dividend on £200 Metropolitan Board of Works Stock, less Income Tax	6	16	4			
				52	9	11
Received from Taylor and Francis :—						
Sales in 1888				33	1	6
Balance due to Treasurer				5	4	0
				£422	17	8

Cr.	£	*s.*	*d.*	£	*s.*	*d.*
Balance due to Treasurer				3	12	3
Excess Subscriptions returned				0	3	0
Bank charges				0	1	6
Taylor and Francis :—						
Proceedings, vol. ix. (parts 1–3)	103	8	0			
Circulars, postage and addressing	12	2	6			
Members' separate copies	21	12	6			
Miscellaneous printing	17	15	1			
Physical Memoirs, vol. i. part 1	54	4	6			
Books	5	12	0			
				214	14	7
Williams and Norgate				13	1	0
Gyde, for Binding				3	4	0
Attendance at Meetings and incidental expenses				7	1	11
Reporting				27	12	6
Brown, for Receipt-book &c.				0	10	9
Petty Cash :—						
Mr. Baily				1	18	0
Balance in Bank				150	15	2
				£422	17	8

Audited and found correct,

RICHARD INWARDS,
GEORGE M. MINCHIN, } *Auditors.*

London January 16th, 1889.

PROPERTY ACCOUNT OF THE PHYSICAL SOCIETY, DECEMBER 31, 1888.

ASSETS.	£	s.	d.
Balance in Bank	150	18	2
Subscriptions due	145	0	0
£400 Furness 4 per cent. Debenture Stock	456	0	0
£575 Midland 4 per cent. Preference Stock	650	0	0
£200 Metropolitan Board of Works Stock	217	0	0
£200 Lancaster Corporation Stock	224	0	0
£254⅔ New South Wales 3½ per cent. Stock	249	7	5
Interest due (estimated dividend on N. S. Wales 3½ per cent.)	8	17	11
	£2101	3	6

LIABILITIES.	£	s.	d.
Balance due to Treasurer	5	4	0
Bills outstanding:—			
Taylor and Francis (one Part of Proceedings &c.)	54	13	8
Petty Cash	1	10	4
Subscriptions anticipated	4	0	0
Balance	2035	15	6
	£2101	3	6

We have examined the above Account, and also the Securities at the Bank, and find the same to be correct.

London, January 16th, 1889.

RICHARD INWARDS,
GEORGE M. MINCHIN, } *Auditors.*

PROCEEDINGS

AT THE

MEETINGS OF THE PHYSICAL SOCIETY

OF LONDON.

SESSION 1889-90.

February 9th, 1889.

Prof. A. W. Reinold, President, in the Chair.

The following were elected Members of the Society:--

Mr. James Wimshurst and Mr. Percy R. Sellon.

The following communication was made:—

"On Physico-Geometrical Models." By Prof. A. S. Herschel, F.R.S.

February 23rd, 1889.

Prof. A. W. Reinold, President, in the Chair.

Mr. Martin H. Kilgour was elected a Member of the Society.

The following communications were made:—

"On Electrical Measurement." By Prof. W. E. Ayrton, F.R.S., and Prof. J. Perry, F.R.S.

"Note on the Measurement of Resistance." By Dr. J. W. Waghorn.

"On the Dimensions of Electromagnetic Units." By Prof. G. F. FITZGERALD, F.R.S.

"Notes on Polarized Light." By Prof. S. P. THOMPSON.

March 9th, 1889.

Prof. A. W. REINOLD, President, in the Chair.

The following were elected Members of the Society :—

Prof. R. THRELFALL and Prof. A. S. HERSCHEL, F.R.S.

The following communication was made :—

"On the Rotation of the Plane of Polarized Light by the Discharge of a Leyden Jar." By Dr. OLIVER LODGE, F.R.S.

March 23rd, 1889.

Prof. A. W. REINOLD, President, in the Chair.

The following were elected Members of the Society :—

Mr. J. W. GRAHAM, Mr. C. CAPITO, Mr. D. G. FITZGERALD, Sir D. GALTON, Mr. W. H. MASSEY, Mr. R. E. CROMPTON, Mr. A. COCKBURN, Sir J. DOUGLASS, Mr. R. W. PAUL, Mr. E. MOYNIHAN, Mr. C. H. W. BIGGS, Mr. C. G. GÜMPEL, Mr. C. E. SPAGNOLETTI, and Mr. F. G. MUDFORD.

The following communications were made :—

"On the Use of Lissajous' Figures for determining Speed of Rotation; and on the Direct Determination of the Vibration Period of a Reed or Tuning-Fork by means of a Morse Receiver." By Prof. J. V. JONES.

"On the Clark Cell as a source of Standard Currents, and its application to the Calibration of Sensitive Long-Range Galvanometers." By Prof. R. THRELFALL and Mr. A. POLLOCK.

"On the Measurement of High Specific Resistances, and on the Measurements of the Resistance of Imperfectly Purified Sulphur." By Prof. R. THRELFALL.

April 13th, 1889.

Prof. A. W. Reinold, President, in the Chair.

The following were elected Members of the Society :—

Mr. P. W. Willans, Mr. J. E. Dowson, Mr. Charles Priest, Mr. C. A. O'Keefe, Mr. W. M. Mordey, and Mr. Alexander Siemens.

The following communications were made :—

"A Lecture-Experiment illustrating the Effect of Heat on the Magnetic Susceptibility of Nickel;"

"An Experiment showing the Effect of Light on Magnetism." Both by Mr. Shelford Bidwell, F.R.S.

"On the Dark Flash as seen in some Lightning Photographs." By Mr. G. M. Whipple.

"On Quartz as an Insulator." By Mr. C. V. Boys, F.R.S.

"On a Refraction-Goniometer." By Mr. A. P. Trotter.

"Apparatus to illustrate Crystal Forms." By Dr. R. J. Anderson.

May 11th, 1889.

Prof. A. W. Reinold, President, in the Chair.

The following were elected Members of the Society :—

Mr. W. Robinson, Prof. V. B. Lewes, and Mr. A. Clegg.

The following communications were made :—

"On an Electrostatic Field produced by varying Magnetic Induction." By Dr. O. Lodge, F.R.S.

"On Electric Radiation, and its Concentration by Lenses." By Dr. O. Lodge, F.R.S., and Dr. J. Howard.

May 25th, 1889.

Prof. A. W. Reinold, President, in the Chair.

Mr. T. A. Lawson was elected a Member of the Society.

The following communications were made :—

"On a Relation existing between the Density and Refraction of Gaseous Elements, and also of some of their Compounds." By the Rev. T. Pelham Dale, M.A.

"On a Water-Spray Influence-Machine." By Mr. GEORGE FULLER.

"Notes on Polarized Light." By Prof. S. P. THOMPSON.

"On the Molecular Weight of Caoutchouc and other Colloid Bodies." By Dr. J. H. GLADSTONE, F.R.S., and Dr. WALTER HIBBERT.

"A Note on Mr. Pelham Dale's paper." By Prof. A. W. RÜCKER, F.R.S.

June 8th, 1889.

Prof. AYRTON, Vice-President, in the Chair.

The following were elected Members of the Society :—

Mr. LUDWIG MOND and Mr. JOHN R. BRITTLE.

The following communications were made :—

"On Intermittent Lightning Flashes." By Dr. H. H. HOFFERT.

"On the Methods of suppressing Sparking in Electro-Magnetic Circuits." By Prof. S. P. THOMPSON.

"On a Shunt-Transformer." By Mr. E. W. SMITH.

"Notes on Geometrical Optics." By Prof. S. P. THOMPSON.

"On the use of the Bi-Quartz in determining the Position of the Plane of Polarization." By Mr. A. W. WARD.

June 22nd, 1889.

Prof. A. W. REINOLD, President, in the Chair.

The following were elected Members of the Society :—

Mr. A. W. WARD and Major CARDEW.

The following communications were made :—

"On some Photographs of Lightning, and of *Black* Electric Sparks." By Mr. A. W. CLAYDEN.

"Researches on the Electrical Resistance of Bismuth." By EDMOND VAN AUBEL.

"On the Expansion with Rise of Temperature of Wires under Pulling Stress." By Mr. J. T. BOTTOMLEY, F.R.S.

November 1st, 1889.

Prof. REINOLD, President, in the Chair.

Mr. T. A. GARRETT was elected a Member of the Society.

The following communications were made :—

"On a new Electric Radiation-Meter;"

"On a Method of driving Tuning-Forks Electrically." Both by W. G. GREGORY, M.A.

"On a Physical Basis for the Theory of Errors." By Mr. C. V. BURTON.

"On the Behaviour of Twisted Strips." By Prof. PERRY, F.R.S.

"On Electrifications due to the Contact of Gases and Liquids." By Mr. J. ENRIGHT.

November 15th, 1889.

Prof. REINOLD, President, in the Chair.

The following were elected Members of the Society :—

Prof. J. MILNE, F.R.S., and Lieut. G. F. EYRE, R.N.

The following communications were made :—

"On Electrifications due to the Contact of Gases and Liquids." By Mr. J. ENRIGHT.

"On the Effect of repeated Heating and Cooling upon the Electrical Resistance of Iron." By Mr. HERBERT TOMLINSON, F.R.S.

December 6th, 1889.

Prof. REINOLD, President, in the Chair.

Mr. J. W. SWAN was elected a Member of the Society.

The following communications were made :—

"On the Electrification of a Steam-Jet." By SHELFORD BIDWELL, M.A., F.R.S.

"Notes on Geometrical Optics." By Prof. S. P. THOMPSON.

"On the Behaviour of Steel under Mechanical Stress." By Mr. C. A. CARUS-WILSON.

January 17th, 1890.

Prof. ADAMS, Vice-President, in the Chair.

The following communications were made :—

"On Electric Splashes." By Prof. S. P. THOMPSON.

"On Galvanometers." By Prof. AYRTON, F.R.S., Mr. T. MATHER, and W. E. SUMPNER, D.Sc.

Annual General Meeting.

February 7th, 1890.

Prof. A. W. REINOLD, President, in the Chair.

The following Report of the Council was read by the Chairman :—

The Council have again the satisfaction of recording an increase in the number of Members, which now exceeds 360. It may be worthy of note that among our Members are about 80 Fellows of the Royal Society.

The Meeting of the Society on Saturday afternoons has long been felt by Members to be inconvenient. Some years ago the Council issued a circular asking for the opinions of Members on this question, and it was ascertained that the Society was nearly equally divided in opinion, the majority in favour of a change being so small that the Council did not feel themselves justified in disturbing the then existing arrangement. In the past year the question was reconsidered, and after carefully taking into account the times of meeting of other Societies whose work is closely connected with our own, and whose Meetings many of our own Members are accustomed to attend, the Council came to the conclusion that the afternoons of the 1st and 3rd Fridays in each month was the only time to which our meetings could be transferred without interfering with the attendance of our Members at the meetings of other Societies. The Council issued a circular asking Members whether they were in favour of, or against, or indifferent to the suggested change; and they received a large

number of replies, giving a very strong majority in favour of the change—the numbers being in favour 129, indifferent 54, against 30. The Council accordingly decided to change the time of meeting to the 1st and 3rd Fridays, at 5 P.M. The change came into effect last November, and is found more convenient than the Saturday afternoons to the Members who attend the meetings, although it has not produced a satisfactory increase in the numbers of those who attend.

During the present Session the second part of the Translations of Foreign Scientific Memoirs has been issued to the Members. This embraces four Memoirs by Prof. Hittorf on the Conduction of Electricity in Gases, and a Memoir by Dr. Puluj on Radiant Electrode Matter, the whole containing 331 pages, with 68 woodcuts and three lithographic plates.

Prof. Threlfall and Mr. Adare, having completed the translation of Van der Waals' memoir on the Continuity of the Liquid and Gaseous States of Matter, referred to in last year's report, the whole of this is now in type. In order to save time, the proofs read for press have been forwarded to Sydney, and as they are returned the work will be printed off, and the Committee confidently expect that it will be issued to the Members during the early part of the present Session.

No further progress has been made with Volta's works during the year, an important public engagement having prevented Mr. Cunynghame from giving any active help. This engagement being now at an end, it may be anticipated that the Society may look for this help, and that the remainder of the work will be ready in the course of the present year. The publication, however, of Van der Waals' memoir is as much as can be looked for at present.

The Council recommend the election of Prof. Hittorf, of Münster, as an Honorary Member of the Society.

The Council have the pleasure of announcing that the following valuable collection of books relating to Electricity has been offered by the Council of the Royal Astronomical Society, and will form a welcome addition to our Library :—

Natural Philosophy.

ADAMS, G. Essay on Electricity. 8vo. 1784.
ALDINI, J. Account of Improvements in Galvanism. 4to. 1803.
BENNET, A. New Experiments on Electricity. 8vo. Derby, 1789.
BROOK, A. Experiments and Remarks on Electricity (&c.). 4to. Norwich, 1789.

CAVALLO, T. Complete Treatise on Electricity. 4th edit. 3 vols. 8vo. 1795.
CAVALLO, T. Treatise on Magnetism. 2nd edit. 8vo. 1795.
CAVALLO, T. Elements of Natural or Experimental Philosophy. 4 vols. 8vo. 1803.
FERGUSON, J. Introduction to Electricity. 8vo. (2 copies.) 1770.
FRANKLIN, B. Experiments and Observations on Electricity. 4to. 1769.
FRANKLIN, B. New Experiments and Observations on Electricity. 3 parts, in 1 vol. 4to. 1760–65.

[*Bound with above Vol.*]

SYMMER (R.). Experiments and Observations on Electricity. 4to. 1760.
PRIESTLEY, J. Account of Henly's Electrometer. 4to. 1773.
HENLY, W. Experiments concerning Lightning Conductors. 1774.
HENLY, W. New Experiments in Electricity. 4to. 1774.
PRIESTLEY, J. Familiar Introduction to Electricity. 1768.

FRANKLIN, B. Philosophical and Miscellaneous Papers. 8vo. 1787.
FRANKLIN, B. Works. 2nd edit. 3 vols. 8vo. [n. d.]
HOLMES. Magneto-Electric Light in Lighthouses. 8vo. [1862.]
LORIMER, J. Essay on Magnetism. 4to. 1795.
MAHON, Lord. Principles of Electricity. 4to. 1779.
MUSSCHENBROEK, P. VAN. Elements of Natural Philosophy. Colson. 2 vols. 8vo. 1744.
PRIESTLEY, J. History and Present State of Electricity. 4to. 1767.
PRIESTLEY, J. Familiar Introduction to the Study of Electricity. 4to. 1768.
SINGER, G. J. Elements of Electricity. 8vo. 1814.
WATSON, W. Sequel to Experiments and Observations on Electricity. 8vo. 1746.
WILKINSON, C. H. Elements of Galvanism. 2 vols. 8vo. 1804.

The following Journals and Books have been received by the Physical Society:—

Newspapers and Magazines:—

Nature.
The Electrical Review.
Engineering.
The Electrician.
The Electrical Engineer.
Electrical Plant.
Ingeniero y Ferretero Español.
Crónica Cientifica.
The Philosophical Magazine.
Beiblätter der Physik und Chemie.
Annalen der Physik und Chemie.
Journal de Physique.

Journals &c. of Societies, British:—

Royal Society, Proceedings.
Society of Arts, Journal.
Institution of Electrical Engineers, Journal.
Mechanical Engineers, Proceedings.
Royal Meteorological Society, Quarterly Journal.
Royal Dublin Society, Proceedings and Transactions.
Middlesex Natural History and Science Society, Transactions.
Philosophical Society of Glasgow, Proceedings.
Philosophical Society of Cambridge, Proceedings and Transactions.
Literary and Philosophical Society of Manchester, Memoirs and Proceedings.
Royal Institution, Proceedings.
Geological Society of Glasgow, Transactions.
Birmingham Philosophical Society, Proceedings.

Journals of Societies, Colonial and Foreign:—

Royal Society of New South Wales, Journal and Proceedings.
Canadian Institute, Proceedings.
New Zealand Institute, Transactions.
Magnetic and Meteorological Observations, Bombay.
Academy of Natural Sciences, Philadelphia, Proceedings.
Physical and Chemical Society of Russia, Journal.
Physical Society of Berlin, Verhandlungen.
Physical Society of France, Séances. Mémoires, Tome iv.
College of Science of Japan, Journal.
Seismological Society of Japan, Transactions.
R. Accad. Torino, Atti and Bulletino.
Cincinnati Society of Natural History, Journal.
Bulletin International de l'Académie des Sciences de Cracovie, Comptes-Rendus.
St. Louis Academy of Science, Transactions.
New York Academy of Science, Transactions.
Ohio Meteorological Bureau, Report.
Bulletino della Societa Ital. di Elettricita.

Books and Papers:—

Calendars of Glasgow University, City of London College, and University of London.
Institution of Civil Engineers, Charter and List of Members.

Society of Chemical Industry, Address of President.

Sur les Propriétés Physiques de la Couche Superficielle libre d'un Liquide.
Sur un genre particulier d'expérience Capillaire.
— G. Van de Mensbrugghe.

Winding of Dynamo Fields.
Report of Missouri Rainfall.
Output of Non-Condensing Steam-Engines.
— Francis E. Nipher.

Sur les Phénomènes Actino-Eléctriques, by M. A. Stoltetow.

Contributions to Canadian Palæontology, Vol. I.

Prace Matematyczno-Fizyczne, Tome II. pt. 1.

The Council regret to have to record the loss by death of three well-known Members of the Society—Dr. James Prescott Joule, Mr. Warren De La Rue, and the Rev. Stephen Joseph Perry.

James Prescott Joule died at Sale, near Manchester, on Friday night, October 11th, 1889, after many years of ill-health. It is admitted by those most competent to judge that his name must be classed alongside of the great original workers in science. Many who have never heard of the name of Dr. Joule have heard of the 'Mechanical Equivalent of Heat,' a phrase, it is to be feared, that to the majority of even intelligent people has little meaning. It is mainly on the determination of this 'Mechanical Equivalent' that the late Dr. Joule's title to fame rests. Joule was born at Salford, Manchester, on Christmas Eve, 1818, so that he has died at a ripe old age. He was so delicate that he could not be sent to school, and so was privately educated at home. Both his father and grandfather were brewers, and young Joule and his brother also entered the business, though James soon became so absorbed in scientific investigations that he practically left the brewery to others. Soon after entering the brewery, about the age of 15, he and his brother were sent by their father to the great chemist Dalton, the propounder of the atomic theory, in order to learn chemistry. One can easily fancy the influence that contact with a mind like Dalton's would have on a youth like Joule. He seems to have been at once caught with the enthusiasm for scientific research, and soon, with the rudest apparatus, began to make experiments of his own, mainly on the chemical constitution of gases and on the relations between chemical action and electricity. One of his first published papers was on a new Electro-Magnetic Engine, which he described at the age of 19. The amount and variety of

his work since then may be inferred from the fact that in the Royal Society List there are the titles of 97 papers, besides over 20 other important papers on researches undertaken by him in conjunction with Sir William Thomson, Sir Lyon Playfair, and Dr. Scoresby. All his papers, revised by himself, have been published by the Physical Society.

In conducting his many experiments in Electromagnetism he had met with "the difficulty, if not impossibility, of understanding experiments and comparing them with one another, which arises in general from incomplete descriptions of apparatus, and from the arbitrary and vague numbers which are used to characterize electric currents . . . . I have therefore determined for my own part to abandon my old quantity numbers and to express my results on the basis of a unit which shall be at once scientific and convenient." He set himself to devise an accurate method of measuring a unit-quantity of electricity, which he took to be the quantity required to decompose nine grains of water. He went on to devise many improvements in galvanometers and to carry out a vast variety of ingenious experiments on the various aspects of electricity, and was thus led in 1841 and 1842 to lay the foundation of a new department of physical science—Thermodynamics, in which so much good work has been done since. Meantime he had been speculating on the conversion of chemical energy into heat; and in 1840, 1841, and 1842 he gave accounts of a number of experiments, all leading up to the great determination of the Mechanical Equivalent of Heat. This value in its earliest form was first announced at the Cork Meeting of the British Association in 1843, in a paper on "The Calorific Effects of Magneto-Electricity, and on the Mechanical Value of Heat." In it, among other things, he said :—"I have lately found experimentally that heat is evolved by the passage of water through narrow tubes. . . . I obtained one degree of heat per pound of water from a mechanical force capable of raising about 770 lb. to the height of one foot. . . . I shall lose no time in repeating and extending these experiments, being satisfied that the grand agents of nature are, by the Creator's fiat, indestructible, and that, whatever mechanical force is expended, an exact equivalent of heat is always obtained." The importance of the announcement it is impossible to overrate. Of course others at home and abroad had been working in the same direction, just as had been the case before Darwin announced his great discovery. But, though at first it was slowly and somewhat grudgingly recognized by his fellow-students,

Joule's discovery made its way surely, and is now recognized as one of the ruling constants of science. Of course he continued to work at, and to perfect, his mechanical equivalent; and in a paper to the Royal Society in 1878 he gave the results of a fresh determination, according to which the quantity of work required to be expended in order to raise the temperature of 1 lb. of water, weighed in vacuum, from 60° to 61° Fahrenheit, is 772·55 foot-pounds of work, at the sea-level, and in the latitude of Greenwich. Joule's great experiment has been repeated by many hands and in many forms and on great scales, but nothing has shaken the truth of the discovery.

The scientific bearings of Joule's great discovery are of the widest and profoundest character; and he is entitled to be placed among the greatest of original workers in pure science. And the practical results which have accrued from his theory have been of the most useful and beneficent kind. They have enabled our engineers to improve their machinery along lines of precision not otherwise attainable; while the amount saved in the application of heat must have amounted to millions. Joule continued his scientific investigations till very near the end, and, varied and important as they were, they cannot be entered into here. He lived the most retired life, partly from weak health and partly from his native modesty; and so it happens that one of the greatest of Darwin's contemporaries was hardly known to the outside world. He was honoured, among those whose opinions alone he valued, at home and abroad. Gold Medals and other marks of appreciation were showered upon him, including those of the Royal Society and Society of Arts. In 1878 Her Majesty, through Lord Beaconsfield, conferred upon him a pension of £200 a year; and few pensions were ever better deserved.

Warren De La Rue, D.C.L., F.R.S., Ph.D., who died on Good Friday, 19th April, 1889, was the son of the late Mr. Thomas De La Rue, and was born in Guernsey in the year 1815. He was educated in Paris, and succeeded his father as head of the Firm of Thomas De La Rue and Co., from which he retired in 1880. He acted as juror and reporter in the Department of Class XXIX. in the Great Exhibition of 1851, was a juror in Class X. of the Paris Exhibition of 1855, and presided over Section B Class XXVIII. of the Exhibition of 1862. In 1860 he went to Spain with the Himalaya Expedition, and was successful in obtaining a series of photographs of the total eclipse of the sun on July 18th. The results formed the subject of the Bakerian Lecture, delivered to the Royal Society

in April 1862. In conjunction with Professor Balfour Stewart and Mr. B. Loewy he published 'Researches in Solar Physics,' founded on observations made at the Kew Observatory under his directions. He also took an active part in making the preparations for the photographic observation of the Transit of Venus in 1874. He established a private Observatory at Cranford, Middlesex; but it was dismantled in 1873, and the instruments presented to the University of Oxford, where they have been employed most successfully by Professor Pritchard in determining, by means of photography, the distance of 61 Cygni and other fixed stars. In 1874 he fitted up a private Physical Laboratory, where, employing a battery of 15,000 chloride-of-silver cells, he, in conjunction with Dr. Hugo Müller, carried on an elaborate series of researches on the electrical discharge. The results of the researches were communicated to the Royal Society and the Académie des Sciences, Paris. He acted for some time as Honorary Secretary of the Royal Astronomical Society, of which he was also President from 1864 to 1866. He was President of the Chemical Society from 1867 to 1869, and again in 1879-88. He also filled the offices of President of the London Institution and Secretary of the Royal Institution. He was a Corresponding Member of the French Académie des Sciences, for the Department of Astronomy; also of the Imperial Academy of Sciences, St. Petersburg, and other Foreign Societies. Three foreign orders were conferred upon him:—Commander of the Legion of Honour, Commander of the Order of St. Maurice and St. Lazare, and Knight of the Order of the Rose, Brazil. He was a Member of this Society almost from its commencement.

Stephen Joseph Perry was born in London on August 26, 1833. After studying at the Catholic College at Douay, he went to Rome to study philosophy, and entered the Order of the Jesuits at the age of twenty. A few years later he went through a special course of Mathematics in Paris; and in 1860 was appointed Professor at Stonyhurst College and Director of the Observatory, which positions he held till his death, with the exception of five years spent in studying theology at St. Beuno's College, North Wales, and in making a Magnetic Survey of France. He was ordained priest in 1866. On April 9, 1869, Father Perry was elected a Fellow of the Royal Astronomical Society, and on June 4, 1874, a Fellow of the Royal Society, and he was one of the original Members of the Physical Society. Among the other Societies with which he was connected as Member or Associate are the Royal Meteorological, the Brussels Academy of Sciences, and the Accademia dei Lincei.

Some idea of the esteem in which astronomers held their departed colleague may be inferred from the fact that on more than one occasion he was a Member of the Council of the Royal Astronomical Society, and was recently elected President of the Liverpool Astronomical Society. The reason for this esteem is not far to seek, for to the qualities of an able and enthusiastic astronomer Father Perry added a readiness to aid all who sought his counsel, and he commanded the respect of all by his unselfishness where astronomy was concerned. To many persons a trip on an astronomical expedition would be a pleasant holiday; to him it was a terrible trial, as he was a martyr to sea-sickness. Notwithstanding this, we find him in 1874 at Kerguelen, and in 1882 at Madagascar, in 1886 in the West Indies, and in 1887 in Russia. Only those who were his companions on these occasions are fully aware of his devotion to Science displayed in voluntarily undergoing so much suffering. His correspondence contains no reference to it; but Mr. Turner, of the Royal Observatory (who went to the West Indies in 1886), in an astronomical journal published this morning, says:—"Father Perry has been one of the most consistent and cheerful sufferers in the cause of Science. He was always a martyr to sea-sickness, but has been a member of more Astronomical Expeditions than any man now living."

Last year he went in charge of the Eclipse Expedition to the Salut Islands, where an attack of dysentery seized him at 3 o'clock on the morning of the eclipse. So rapidly weak did Father Perry become that he had to be assisted to the Observatory at the Salut Islands. Feeling uncertain as to the course his illness would take, Father Perry requested and received the last Sacrament from the resident priest before her Majesty's ship 'Comus' left the island for Demerara. Father Perry's worst fears, however, were realized, and before the vessel arrived at Demerara the indefatigable astronomer departed this life, on December 12th, 1889.

His solar work during the last ten years formed the subject of a lecture at the Royal Institution on May 24. It may be divided into two classes—drawings and spectroscopic observations. For the drawings an image of the sun $10\frac{1}{2}$ inches in diameter was projected on a sheet of drawing-paper affixed to a sketch-board carried by the telescope, and all markings on the sun traced. The drawing finished, the chromosphere and prominences were examined with the spectroscope. About 250 drawings were made every year from 1880.

The adoption of the Report was proposed, and carried unanimously.

The Society then elected Professor W. Hittorf an Honorary Member of the Society.

The election of Officers and other Members of Council then took place, and the new Council was constituted as follows:—

President.—Prof. W. E. Ayrton, F.R.S.

Vice-Presidents who have filled the office of President.—Dr. J. H. Gladstone, F.R.S.; Prof. G. C. Foster, F.R.S.; Prof. W. G. Adams, M.A., F.R.S.; Sir William Thomson, D.C.L., LL.D., F.R.S.; Prof. R. B. Clifton, M.A., F.R.S.

Vice-Presidents.—Dr. E. Atkinson; Walter Baily, M.A.; Shelford Bidwell, M.A., LL.B., F.R.S.; Prof. S. P. Thompson, D.Sc.

Secretaries.—Prof. J. Perry, F.R.S.; T. H. Blakesley, M.A., M.Inst.C.E.

Treasurer.—Prof. A. W. Rücker, M.A., F.R.S.

Demonstrator and Librarian.—C. Vernon Boys, F.R.S.

Other Members of Council.—W. H. Coffin; Sir John Conroy, Bart., M.A.; Conrad W. Cooke; Major-General E. R. Festing, R.E., F.R.S.; Prof. J. V. Jones, M.A.; Prof. O. Lodge, D.Sc., F.R.S.; Prof. W. Ramsay, Ph.D., F.R.S.; W. N. Shaw, M.A.; H. Tomlinson, B.A., F.R.S.; G. M. Whipple, D.Sc.

Votes of thanks were passed to the Lords Committee of the Council on Education; to the Officers; and to the Auditors.

JANUARY 1st, 1889, TO DECEMBER 31st, 1889.

Dr.	£ s. d.	£ s. d.
Balance in Bank, January 1, 1889		150 18 2
Entrance-Fees	29 0 0	
Subscriptions for 1882	1 0 0	
„ 1883	1 0 0	
„ 1884	2 0 0	
„ 1885	2 0 0	
„ 1886	2 0 0	
„ 1887	4 0 0	
„ 1888	18 0 0	
„ 1889	131 0 0	
„ 1890	1 0 0	
Excess Subscriptions	1 15 0	
Eleven Life Subscriptions	110 0 0	
		302 15 0
One year's Dividend on £400 Furness 4 per cent. Debenture Stock, less Income Tax	15 12 0	
One year's Dividend on £575 Midland 4 per cent. Preference Stock, less Income Tax	22 8 6	
One year's Dividend on £200 Lancaster Corporation Stock, less Income Tax	7 16 0	
One year's Dividend on £200 Metropolitan Board of Works Stock, less Income Tax	6 16 8	
Two years' Dividends on £254⅔ New South Wales 3½ per cent. Stock	17 6 7	
		69 19 9
Received from Taylor and Francis :—		
Sales in 1889		20 14 11
Balance due to Treasurer		7 5 6
		£551 13 4

Cr.	£ s. d.	£ s. d.
Balance due to Treasurer		5 4 0
Excess Subscriptions returned		1 15 0
Taylor and Francis :—		
Proceedings, vol. ix. (part 4); vol. x. (parts 1 & 2)	124 1 0	
Circulars, addressing and posting	12 10 6	
Members' separate copies	24 19 0	
Miscellaneous printing	28 0 4	
Physical Memoirs	101 13 6	
		291 4 4
Williams and Norgate		4 5 0
Haggard		13 11 6
Attendance at Meetings and incidental expenses		7 7 6
Reporting		14 1 8
Petty Cash :—		
Prof. Rücker		2 10 4
Balance in Bank		211 14 0
		£551 13 4

Examined and found correct,

January 30th, 1890.

GEORGE M. MINCHIN,
ALFRED H. FISON, } *Auditors.*

PROPERTY ACCOUNT OF THE PHYSICAL SOCIETY, DECEMBER 31, 1889.

ASSETS.	£	s.	d.
Balance in Bank	211	14	0
Subscriptions due	157	0	0
£400 Furness 4 per cent. Debenture Stock	456	0	0
£575 Midland 4 per cent. Preference Stock	650	0	0
£200 Metropolitan Board of Works Stock	217	0	0
£200 Lancaster Corporation Stock	224	0	0
£254⅜ New South Wales 3½ per cent. Stock	249	7	5
	£2165	1	5

LIABILITIES.	£	s.	d.
Balance due to Treasurer	7	5	6
Bills outstanding:—			
Gyde	1	18	0
Subscriptions anticipated	1	0	0
Balance	2154	17	11
	£2165	1	5

Examined, and found correct.

January 30th, 1890.

GEORGE M. MINCHIN,
ALFRED H FISON, } *Auditors.*